Canadian Association of Physicists
2002 Prize Exam

This is a three hour exam. National ranking and prizes will be based on a student's performance on both sections A and B of the exam. However, performance on the multiple choice questions in part A will be used to determine whose written work in part B will be marked for prize consideration by the National Committee. The questions in part B of the exam have a range of difficulty. Do be careful to gather as many of the easier marks as possible before venturing into more difficult territory. If an answer to part (a) of a question is needed for part (b), and you are not able to solve part (a), assume a likely solution and attempt the rest of the question anyway. No student is expected to complete this exam and parts of each problem may be very challenging.
Non-programmable calculators may be used. Please be very careful to answer the multiple choice questions **on the answer card/sheet** provided; most importantly, write your solutions to the three written problems on **separate** sheets as they will be marked by people in different parts of Canada. Good luck.

Data

Speed of light	$c = 3.00 \times 10^8$ m/s
Gravitational constant	$G = 6.67 \times 10^{-11}$ N·m^2/kg^2
Acceleration due to gravity	$g = 9.80$ m/s^2
Fundamental charge	$e = 1.60 \times 10^{-19}$ C
Mass of electron	$m_e = 9.11 \times 10^{-31}$ kg
Mass of proton	$m_p = 1.673 \times 10^{-27}$ kg
Coulomb's constant	$1/4\pi\epsilon_o = 8.99 \times 10^9$ J·m/C^2
Permeability of free space	$\mu_o = 4\pi \times 10^{-7}$ N/A^2
Speed of sound in air	$v_s = 343$ m/s
Index of refraction of water	$n_{\rm w} = 1.33$
Energy conversion	1 eV=1.6×10^{-19} J

Part A: Multiple Choice

Question 1

A little girl is holding a helium-filled balloon on a string while riding in a closed elevator going down a very tall building at constant speed. The elevator shaft is maintained under vacuum. Suddenly the elevator cable snaps, sending the elevator into free fall. In her surprise the girl lets go of the string. She is even more surprised to see

(a) the balloon rising.

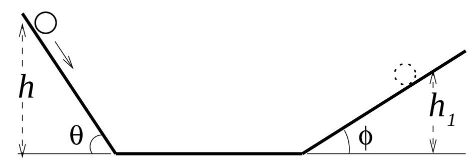
(b) the balloon floating downward.

(c) the balloon remaining stationary.

(d) the balloon bouncing slowly between the floor and the ceiling.

Question 2

A demonstration apparatus sits on a table in the lab. It consists of a metal track (shown as a thick solid line in the figure below) along which a perfectly spherical marble can roll without slipping. In one run, the marble is released from rest at a height h above the table on the left section, rolls down one side and then up the other side without slipping, briefly stopping when it has reached h_1. Assuming the table to be horizontal and neglecting air drag as well as any energy loss due to rolling,



(a) $h_1 < h$ always, because the friction that keeps the ball rolling must dissipate energy.

(b) $h_1 = h$, since total mechanical energy is conserved.

(c) $h_1 = h$ only if tilt angles ϕ and θ are equal.

(d) $h_1 = h$ only if tilt angle ϕ is larger than θ.

Question 3

In the movie *Jurassic Park*, as a jeep is being pursued by a dinosaur, one can see the animal in the jeep's side rear-view mirror on which is printed: "Objects in mirror are closer than they appear." Then,

(a) the mirror is flat.

(b) the mirror is concave.

(c) the mirror is convex.

(d) the mirror could be either convex or concave.

Question 4

A lump of sticky chewing-gum with mass m moves at speed v toward another lump of mass M at rest. They collide and stick together, both moving at speed V. Nothing else is known about the conditions under which the collision takes place. Which of the following statements is the most correct?

(a) Neither total kinetic energy nor total linear momentum can be conserved.

(b) This is an elastic collision in which both total kinetic energy and total linear momentum are conserved; the final speed is $V = v/2$.

(c) This is an inelastic collision, and in such collisions, total linear momentum is always conserved; the final speed is $V = v/4$.

(d) This is an inelastic collision in which total linear momentum is conserved, provided no external force can deliver an impulse to the system during the collision.

Question 5

According to the Archimedes principle,

(a) a body immersed in a fluid experiences an upward force equal to the weight of the fluid it displaces.

(b) a body immersed in a fluid experiences an upward force equal to the weight of the object.

(c) a body immersed in a fluid displaces an amount of fluid equal to the volume of the body.

(d) a body immersed in a liquid floats only if its density is smaller than that of the liquid.

Question 6

A guitar string of length L, with both of its ends fixed, is vibrating, producing a sound composed of a fundamental frequency and its integer multiples (harmonics). To emphasize all the even-numbered harmonics at the expense of the odd-numbered ones, the furthest distance away from either end where one should hold down the string is

(a) $L/2$. (b) $L/3$.
(c) $L/4$. (d) $L/5$.

Question 7

A sinusoidal wave is moving along a string of uniform density. If you double the frequency of the wave,

(a) the wavelength of the wave doubles.

(b) the speed of the wave doubles.

(c) the speed of the wave remains about the same.

(d) the period of the wave doubles.

Question 8

Two point-charges, each with a charge of +1 μC, lie some finite distance apart. On which of the segments of an infinite line going through the charges is there a point, a finite distance away from the charges, where the electric potential is zero, assuming that it vanishes at infinity?

(a) Betweeen the charges only.

(b) On either side outside the system.

(c) Impossible to tell without knowing the distance between the charges.

(d) Nowhere.

Question 9

Standing on the ground, you are holding with your hand a string from which is suspended a stone. The reaction force, associated—in the sense of Newton's 3$^{\text{rd}}$ law—with the force of Earth's gravity on the stone, is the force exerted by

(a) the string on your hand. (b) the stone on the string.
(c) your body on the Earth. (d) the stone on the Earth.

Question 10

At the shallow end of a swimming pool, the water is 70.0 cm deep. The diameter of the cone of light emerging from the water into the air above, emitted by a light source 10.0 cm in diameter at the bottom of the pool and measured by an observer on the edge of the pool 2.50 m away, is'

(a) 1.60 m. (b) 1.70 m.
(c) 1.75 m. (d) 1.80 m.

Question 11

Of two quantities of water and steam, both at 100°C and having the same mass, in immediate contact with the same area of your skin, which is likeliest to cause a more severe burn?

(a) The steam. (b) There is no difference.
(c) The water. (d) It depends on the steam's pressure.

Question 12

A varying electric current running through a coiled wire induces an emf across the coil. The voltage measured by a voltmeter across the terminals of the coil

(a) is smaller or larger than the emf, depending on how fast the current changes.

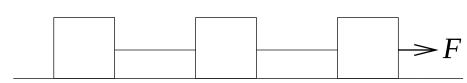
(b) is larger than the emf.

(c) is equal to the emf.

(d) is smaller than the emf.

Question 13

Three blocks of identical mass are connected by strings as shown below. The whole system is accelerated to the right along a frictionless, horizontal surface by a force F. The net force acting on the middle block is



(a) zero. (b) F.
(c) $2F/3$. (d) $F/3$.

Question 14

A spaceship is moving straight toward Earth with its engine turned off. Let M and m be the masses of the Earth and of the spaceship, respectively, with R the distance from the spaceship to the centre of the Earth. As a result of the gravitational force of the Earth, in moving from position 1 to position 2, the kinetic energy of the spaceship increases by

(a) $\dfrac{GMm(R_2 - R_1)}{R_1 R_2}$. (b) $\dfrac{GMm(R_1 - R_2)}{R_1 R_2}$.
(c) $\dfrac{GMm(R_2 - R_1)}{R_1^2}$. (d) $\dfrac{GMm(R_1 - R_2)}{R_1^2 R_2^2}$.

Question 15

Two masses, M_{a} and M_{b}, attached to the end of springs a and b, respectively, undergo simple harmonic motion on a horizontal, frictionless surface. If the periods of motion are identical, and if $M_{\mathrm{b}} = 2M_{\mathrm{a}}$, the amplitudes A of oscillation of the two masses are related by

(a) $A_{\mathrm{b}} = A_{\mathrm{a}}/\sqrt{2}$. (b) $A_{\mathrm{a}} = A_{\mathrm{b}}/4$.
(c) $A_{\mathrm{a}} = A_{\mathrm{b}}$. (d) none of the above.

Question 16

Consider a circuit which contains a battery and a single resistance R. If a second resistance is added in parallel with R,

(a) the voltage across R will decrease.
(b) the current through R will increase.
(c) the total current drawn from the battery will increase.
(d) the power dissipated in R will decrease.

Question 17

Starship *Enterprise* is in a circular orbit of period 300 minutes around planet Thera at an altitude of 1000 km. A Klingon spaceship then enters another circular orbit around Thera at an altitude of 19000 km. Thera has a radius of 5000 km. The Klingon spaceship circles Thera in

(a) 600 minutes. (b) 1200 minutes.
(c) 1800 minutes. (d) 2400 minutes.

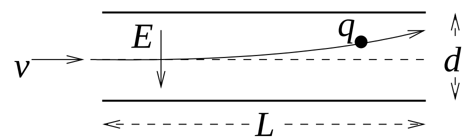
Question 18

A simplified bola consists of two (instead of the usual three) small but heavy balls connected to a common point by identical lengths of string. To launch it, you hold one of the balls overhead and swing the other ball in a horizontal circle about your wrist. You then let go of the bola which very soon starts rotating around its centre of mass, as shown below. Which one of the following statements holds true when the rotation axis changes from (1) to (2)?

(a) Angular momentum is the same, but angular speed increases.
(b) Angular momentum and angular speed remain the same.
(c) Angular momentum is the same, but angular speed decreases.
(d) Angular momentum and angular speed both increase.

Question 19

In an ink-jet printer, an ink droplet of mass m is given a negative charge q by a computer-controlled charging unit, and then enters at speed v the region between two deflecting parallel plates of length L separated by distance d (see figure below). All over this region exists a downward electric field which you can assume to be uniform. Neglecting the gravitational force on the droplet, the maximum charge that it can be given so that it will not hit a plate is most closely approximated by



(a) $\dfrac{mv^2E}{dL^2}$. (b) $\dfrac{mv^2d}{EL^2}$.
(c) $\dfrac{md}{E(vL)^2}$. (d) $\dfrac{m(vL)^2}{Ed}$.

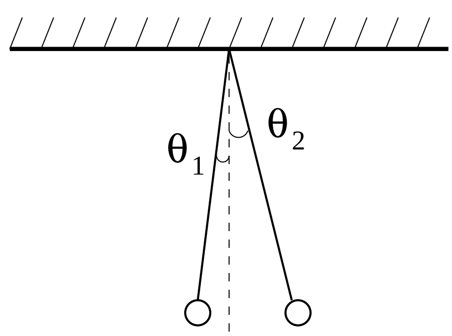
Question 20

Four very long straight wires carry equal electric currents in the $+z$ direction. They intersect the x–y plane at $(x, y) = (-a, 0)$, $(0, a)$, $(a, 0)$, and $(0, -a)$. The magnetic force exerted on the wire at position $(-a, 0)$ is along

(a) $+y$. (b) $-y$.
(c) $+x$. (d) $-x$.

Question 21

Two masses, both equal to 100 g, are suspended at the ends of identical light strings of length $\ell = 1.0$ m, attached to the same point on the ceiling (see figure). At time $t = 0$, they are simultaneously released from rest, one at angle $\theta_1 = 10°$, the other at angle $\theta_2 = 20°$ from the vertical. The masses will collide



(a) at $\theta = 0.0°$, 0.50 s later.
(b) at $\theta = 5.0°$ to the right of the vertical, 0.16 s later.
(c) at $\theta = 0.0°$, 0.13 s later.
(d) at $\theta = 5.0°$ to the right of the vertical, 0.10 s later.

Question 22

Two trains are moving toward each other on two parallel tracks at the same speed with respect to the ground. When the whistle of one train blows, the frequency of the sound heard by a passenger on the other train will be

(a) greatest if the air is still.

(b) greatest if a wind blows in the same direction and at the same speed as the other train.

(c) greatest if a wind blows in the opposite direction and at the same speed as the other train.

(d) will not depend on whether there is a wind blowing.

Question 23

In an African wildlife reserve your jeep is travelling at a *constant* speed of 105 km/hr *alongside* a cheetah running at full speed. The animal, frightened by the jeep's noise, keeps at a constant distance of 60 m from the vehicle. Not having paid much attention to the surrounding landscape, you suddenly realize that your jeep is passing the very same spot where it was 60 seconds before. The speed of the cheetah is

(a) 105 km/hr. (b) 94 km/hr.

(c) 99 km/hr. (d) 82 km/hr.

Question 24

A jet-fighter pilot, wishing to experience weightlessness conditions, enters a parabolic trajectory closely approximating that of a projectile in free-fall. His aircraft must fly at a minimum speed of 400 km/hr to keep control, and he should not exceed 600 km/hr so as not to suffer a black-out when he pulls out of the trajectory at the end. There is no wind and air resistance can be compensated by the engine. In order to be weightless as long as possible, he should enter the parabola at an upward angle (with respect to the horizontal ground) of

(a) $33.7°$. (b) $41.8°$.

(c) $56.3°$. (d) $48.2°$.

Question 25

Which one of the following statements is **not** true? The phase difference between two light waves can change

(a) by reflection.

(b) by the waves travelling along paths of different lengths.

(c) by the waves travelling through media that have a different refraction index.

(d) by being transmitted through a boundary between two media with different refraction indices.

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Part B

Problem 1

Passing a car on a two-lane road requires more caution as speeds increase. Apart from maintaining a safe distance between vehicles in the same lane, there should also be enough space between oncoming vehicles while the passing manœuvre takes place.

Take all cars initially, travelling at the same constant speed, to have a length 4.2 m. Suppose further that a passing manœuvre starts with the passing car at least 3.0 car lengths behind the back of the slower car, and that it ends when the driver has pulled in with the back of her car at least 3.0 car lengths ahead of the slower car (no fish-tailing!). The passing car takes 4.0 s at constant acceleration to reach its final passing speed, which it keeps thereafter. This final speed is 11 km/hr greater than the steady speed of the slower car.

(a) Calculate the total time it takes for such a passing manœuvre.

(b) Now suppose that at the instant the passing driver has reached her overtaking speed, she notices 500 m ahead (from the front of her car) a very large truck coming toward her. The speed limit for trucks on that road, which we shall assume the truck driver respects, is 90 km/hr. The slower car is driving at 100 km/hr. Otherwise, the same conditions as in part (a) apply. We assume that for minimal safety she should have completed her manœuvre at least 2.0 s before the truck and her pass each other, But she cannot abort the manoeuvre without braking unless the front of her car is at least 2.0 m behind the back of the slow car, and she cannot brake safely because another car right behind her is also trying to pass the slow car. Given that it takes her 1.6 s to react after she first sees the truck, discuss whether it is safe for her to continue, or whether she should pull back behind the slower car (if she can!). If it turns out that she is in trouble, could she continue accelerating at the same rate up to a higher speed not exceeding 120 km/hr and pass safely while still pulling in three car lengths in front of the slow car?

Problem 2

In Poe's famous horror tale, *The Pit and the Pendulum*, the hero, who finds himself strapped flat to a horizontal surface, notices an almost motionless pendulum hanging from the ceiling 12 m above him. After some time, he also realizes that the pendulum is descending very slowly toward him with, according to the story, the amplitude of the swings and the speed at the lowest point in each swing *both increasing inexorably*. What is swinging is in fact a small razor-edged crescent of steel which will eventually sweep across the man's heart.

Let's analyze this terrifying situation more closely to see if Poe's spell-binding description actually makes physical sense. To do this, we must make some assumptions which will not affect our final conclusion. The blade of mass m is suspended on the end of a massless cable of length l and is executing simple harmonic motion. Assume that the blade is also slowly lowered in very small increments of l which occur only when it goes through its lowest position in each swing, at $\theta = 0°$, where θ is the angular position with respect to the vertical.

(a) Explain in one short sentence why the angular momentum of the razor at $\theta = 0°$ remains constant all through the descent.

(b) Let the initial length of the cord be l_0 and the initial maximum speed of the razor be v_0. Obtain an algebraic expression for v as a function of l.

(c) Derive an algebraic expression for the kinetic energy of the pendulum at $\theta = 0°$ as a function of l.

(d) Make a reasonable simplyfing assumption about the total mechanical energy. Find (always as a function of l) the maximum gravitational potential energy acquired by the pendulum in a given swing.

(e) Find the maximum angular displacement of the razor as a function of l.

(f) Discuss the accuracy of Poe's description of the descent of the pendulum.

Problem 3

One fine afternoon, as his family watches from some distance, a man walking barefoot on moist ground near a tower supporting electric transmission lines suddenly stops and collapses. His relatives call in emergency services, but these arrive too late to prevent the man's death.

The autopsy reveals that the man's heart went into ventricular fibrillation, a fatal condition if not stopped very quickly. His relatives believe that he was electrocuted by some stray electric current from the tower and sue the power company. The court orders a forensic investigation to determine if the company is responsible for the victim's death.

The investigation finds that there was indeed an electrical fault at the time the incident occurred: for about 1.0 s, an electric current of 100 A leaked into the ground from a vertical conducting rod whose rounded tip was just below ground, located 10 m away from where the man was standing. The resistivity of the moist ground was about 100 $\Omega \cdot \mathrm{m}$.

(a) Assuming that the current spread uniformly into the ground from the rounded tip, find the magnitudes of the current density, J (in A/m^2), and of the electric field, E, as a function of the distance r from the tip of the rod. If you don't know the relationship between J and E, you can derive it from Ohm's law and a bit of dimensional analysis.

(b) Calculate E and J at the victim's location.

(c) Under any suitable approximation, obtain a reasonable estimate of the potential difference between the man's feet.

(d) It is reasonable to suppose that the current went up one leg of the man, across the torso (and the heart), and down through the other leg. The commonly accepted values for the resistance of a leg is 300 Ω and that of a torso 1000 Ω in these conditions. From these data, estimate the current across the victim's torso.

(e) Using your expression for $E(r)$, obtain the potential difference, ΔV, between the tip of the rod and a point situated at distance r. Take the tip of the rod to have radius a. If the integral is a problem, you can still get the answer by noticing the similarity between $E(r)$ and the field of a well-known charge configuration.

(f) Currents ranging from 0.10 A to 1.0 A across a torso can trigger fibrillation in the heart. Could the accident have been caused by the current that leaked from the rod?

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