Canadian Association of Physicists
1997 Prize Exam

This is a three hour exam. National ranking and prizes will be based on a student's performance on both sections A and B of the exam. However, performance on the multiple choice questions in part A will be used to determine whose written work in part B will be marked for prize consideration by the National Committee. The questions in part B of the exam have a range of difficulty. Please be careful to gather as many of the easier marks as possible before venturing into more difficult territory. In some cases an answer to part (a) of a question is needed for part (b). Should you not be able to solve part (a), assume a likely solution and attempt the rest of the question anyway. No student is expected to completely finish this exam and parts of each question are very challenging.

Non-programmable calculators may be used. Please be very careful to answer the multiple choice questions on the answer card/sheet provided to you and, most importantly, write your solutions to the three written problems on separate sheets as they will be marked by different people in different parts of Canada. Good luck.

Data

Speed of light	$c = 3.00 \times 10^8$ m/s
Gravitational constant	$G = 6.67 \times 10^{-11}$ Nm^2/kg^2
Radius of Earth	$R_E = 6.38 \times 10^6$ m
Radius of Moon	$R_M = 1.70 \times 10^6$ m
Mass of Earth	$M_E = 5.98 \times 10^{24}$ kg
Mass of Moon	$M_M = 7.35 \times 10^{22}$ kg
Mass of Sun	$M_S = 1.99 \times 10^{30}$ kg
Radius of Moon's orbit	$R_{EM} = 3.84 \times 10^8$ m
Radius of Earth's orbit	$R_{ES} = 1.50 \times 10^{11}$ m
Acceleration due to gravity	$g = 9.81$ m/s^2
Fundamental charge	$e = 1.60 \times 10^{-19}$ C
Mass of electron	$m_e = 9.11 \times 10^{-31}$ kg
Mass of proton	$m_p = 1.673 \times 10^{-27}$ kg
Mass of neutron	$m_n = 1.675 \times 10^{-27}$ kg
Planck's constant	$h = 6.63 \times 10^{-34}$ Js
Coulomb's constant	$1/4\pi\epsilon_o = 8.99 \times 10^9$ Nm^2/C^2
Permeability of free space	$\mu_o = 4\pi \times 10^{-7}$ N/A^2
Speed of sound in air	$v_s = 340$ m/s
Boltzmann constant	$k = 1.38 \times 10^{-23}$ J/K
Absolute zero	$T = 0$ K, $T = -273°$C

Part A: Multiple Choice

Question 1

Two astronauts, each of mass 75 kg, are floating next to each other in space, outside the space shuttle. One of them pushes the other through a distance of 1 m (an arm's length) with a force of 300 N. What is the final relative velocity of the two?

(a) 2.0 m/s
(b) 2.83 m/s
(c) 4.0 m/s
(d) 16.0 m/s

Question 2

Twelve identical resistors (each of resistance R) are placed along the 12 edges of a cube. What is the effective resistance between opposite corners of the cube?

(a) $2R/3$
(b) $5R/6$
(c) R
(d) $12R$

Question 3

A heavy wooden beam of length L lies on the ground. You must carry it with a friend who happens to be twice as strong as you are (she can carry twice as much weight with the same effort). If you pick up the beam at one end, at what distance from your end should your friend carry it in order to be fair?

(a) $2L/3$
(b) $3L/5$
(c) $3L/4$
(d) L

Question 4

A car travelling at 80 km/h needs a braking distance of 50 m to come to a complete stop. What braking distance should the same car need if its initial speed were 160 km/h? You may assume that the same braking force can be applied in both cases.

(a) 70.7 m
(b) 100 m
(c) 141 m
(d) 200 m

Question 5

What would the length of the earth's day be if a pendulum suspended at some point along the equator did not swing (i.e., when it is pulled from its equilibrium point, it does not swing down again)?

(a) 24 hr
(b) 1.2 hr
(c) 1.4 hr
(d) 2.4 hr

Question 6

Two electrons are a certain distance apart from one another. What is the order of magnitude of the ratio of the electric force between them to the gravitational force between them?

(a) 10^8:1

(b) 10^{28}:1

(c) 10^{31}:1

(d) 10^{42}:1

Question 7

A balloon rising vertically with a speed of 5 m/s releases a sand bag at an instant when it is 20 m above the ground. How long after its release does it take for the sand bag to hit the ground? You may ignore air resistance.

(a) 2.01 s

(b) 2.59 s

(c) 2.81 s

(d) 3.01 s

Question 8

A cord is tied to a pail of water and the pail is swung in a vertical circle of radius 1 m. What must the minimum velocity of the pail be at its highest point so that no water spills out?

(a) 3.1 m/s

(b) 5.6 m/s

(c) 20.7 m/s

(d) 100.5 m/s

Question 9

An object of mass 5 kg hangs from a spring and oscillates with a period of 0.5 s. By how much will the equilibrium length of the spring be shortened when the object is removed.

(a) 0.75 cm

(b) 1.50 cm

(c) 3.13 cm

(d) 6.20 cm

Question 10

A certain organ pipe produces a sound of frequency 440 Hz in air. If the pipe is filled with helium at the same temperature and pressure, what frequency will be produced? Sound travels three times faster in helium than it does in air.

(a) 147 Hz

(b) 440 Hz

(c) 880 Hz

(d) 1320 Hz

Question 11

A ladder of length 10 m and mass 20 kg (and with a uniform mass distribution) leans against a slippery vertical wall. The ladder makes an angle of 30° with respect to the vertical. Friction between the ladder and the ground prevents it from sliding downwards. What is the magnitude of the force exerted on the ladder by the wall?

(a) 0 N

(b) 0.57 N

(c) 5.7 N

(d) 57 N

Question 12

A ball of mass 1.0 kg is whirled on the end of a string in a horizontal circle of radius 1.5 m and with a constant speed of 2.0 m/s. The work done on the ball by the tension in the string in one complete revolution is,

(a) 0 J

(b) 2.7 J

(c) 8.0 J

(d) 25.1 J

Question 13

A source of sound emitting a note of constant frequency moves in a horizontal circle at constant speed. A distant observer hears a note which fluctuates in pitch over a range of 20 Hz once every second. If the rate of rotation of the source is doubled, the observer would hear a fluctuation in pitch,

(a) greater than 20 Hz once every second.

(b) of 20 Hz twice every second.

(c) less than 20 Hz once every two seconds.

(d) greater than 20 Hz twice every second.

Question 14

Four identical charges are located at the corners of a square. The electric field is zero at,

(a) any of the four corners of the square.

(b) any of the mid-points of the sides of the square.

(c) the centre point of the square.

(d) at no point within or on the square.

Question 15

The unit of electric charge may be expressed as,

(a) ampere·newton·meter / watt
(b) ampere·volt
(c) ampere / second
(d) ampere·ohm

Question 16

A rectangular coil of wire rotates about an axis which is perpendicular to a uniform magnetic field at a steady rate. Consider the instant when the plane of the coil is parallel to the magnetic field lines. At that instant the induced electromotive force is,

(a) minimum.
(b) maximum.
(c) zero.
(d) constant, the same at all times.

Question 17

An airplane flies from a town A to a town B when there is no wind and takes a time T_o for a return trip. When there is a wind blowing in a direction from town A to town B, the plane's time for a similar return trip, T_w, would satisfy,

(a) $T_o > T_w$
(b) $T_o < T_w$
(c) $T_o = T_w$
(d) the result depends on the distance between the towns.

Question 18

You have an ample supply of 3 μf capacitors. What is the minimum number of these capacitors required to make a circuit with an equivalent capacitance of 2.25 μf.

(a) 3
(b) 4
(c) 5
(d) 6

Question 19

Three 100 W light bulbs are connected in series to a 120 volt power source. If two of the light bulbs are replaced by 60 W bulbs, the brightness of the remaining 100 W bulb is,

(a) brighter than it was before.
(b) dimmer than it was before.
(c) the same brightness as it was before.
(d) will not illuminate.

Question 20

The earth orbits the sun in an elliptical path. Of the following statements, which are true?

I The earth's orbital speed is constant.
II The angular momentum of the earth with respect to the sun is a constant.
III The force acting on the earth due to the sun is a constant.
IV The earth's orbital speed is faster in the spring and fall than it is in the summer and winter.

(a) I only.
(b) II only.
(c) IV only.
(d) II and IV only.

Question 21

A ball is thrown upwards into the air. Taking into account air resistance, the forces acting on the ball while in upwards flight are,

(a) a decreasing upwards force and a constant downwards force.
(b) a decreasing upwards force and a decreasing downwards force.
(c) a decreasing downwards force.
(d) an increasing downwards force.

Question 22

A bullet travelling at 500 m/s and with a mass of 100 g collides and remains embedded in a small lead weight of mass 5 kg. The weight is suspended from a fixed support by a cord of length 5 m and may swing freely. If the weight was initially at rest, what is the period of oscillation of the system immediately after the collision and how does it change as the amplitude of the oscillation decays due to air resistance?

(a) 4.5 s and constant in time.
(b) 4.5 s and increasing in time.
(c) 0.7 s and constant in time.
(d) none of the above.

Question 23

A peculiar force acting upon an object has the equation $F = -cx^3$ where x is the displacement of the object from some origin and $c = 1\ \text{N/m}^3$. What is the magnitude of the work done by this force on the object as the object moves from $x = -2$ m to $x = +2$ m?

(a) 16 J
(b) 8 J
(c) 4 J
(d) 0 J

Question 24

With what minimum speed, with respect to the earth, must an object be launched from the earth so as to escape the gravitational pull of the solar system? You may Ignore the effect of the earth, moon and other planets.

(a) 5.6×10^3 m/s
(b) 11.2×10^3 m/s
(c) 12.2×10^3 m/s
(d) 42.1×10^3 m/s

Question 25

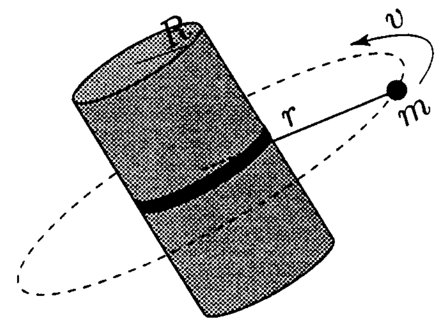
Sound waves from deep within a room and of frequency 440 Hz are incident upon a tall, narrow, and open window. An observer outside is far from the window (at least much farther from the window than the window is wide). The line from the observer to the window makes an angle of 30° with respect to the normal to the plane of the window. Shutters begin to close so that the width of the window is decreasing at 6 m/s. The observer hears,

(a) a 440 Hz note beating at 4.29 Hz.
(b) a 440 Hz note beating at 3.88 Hz.
(c) a 440 Hz note steadily decreasing in intensity.
(d) initially a 440 Hz note with a steadily increasing frequency.

Part B

Question 1

A massive satellite in orbit (*i.e.* in a weightless environment) has a rotating cuff that allows a small mass m on the end of a cord to circle about the satellite at a radius r with a speed v.



The initial speed of the mass is v_o and the initial radius of its circular path is r_o. A mechanism within the cuff allows the cord to be drawn in so that the radius of revolution for the mass decreases. The centre of revolution for the mass remains constant and you may assume that the satellite is much, much more massive than m.

(a) Find the initial tension T_o in the string.

(b) Suppose that the cord begins to be drawn in at a constant rate (r decreases uniformly from its initial value r_o). Express the speed v of the mass as a function of r, r_o and of its initial speed v_o.

(c) If the cord snaps beyond a maximum tension $T_{\max}$ and the radius of the satellite is R, what constraint must be satisfied in order for the mass to be brought within the satellite?

(d) What amount of work is necessary to bring the mass from radius r_o to radius R?

Question 2

A solid metal sphere of radius 2 cm carries a net negative charge $-Q$. The air around the sphere will hence be subjected to an electric field. If an electric field in excess of 3.0×10^6 V/m is applied across an "air molecule", the molecule will ionize. This will appear as sparking and is called "break down". You may assume spherical symmetry in this problem (although in actuality, sparking will break this symmetry).

(a) Sketch the electric field lines within and around the charged sphere and show on your diagram how the excess charge on the sphere is distributed.

(b) What is the minimum charge that the sphere must have to cause the break down of air near its surface?

(c) It requires about 1.6×10^{-18} J of energy to ionize an air molecule. Consider a free electron near the surface of the sphere carrying this minimum charge as in part (b). What radial distance must the electron travel before attaining a kinetic energy equal to the ionization energy for an air molecule? An electron typically travels only 1.0×10^{-6} m before colliding with an air molecule. Is it possible for our accelerated electron to cause ionization of air molecules away from the surface of the sphere?

(d) What is the potential (in units of V) on the surface of the sphere when break down just occurs?

Question 3

A spherical balloon of radius $R = 5$ cm has within it a small, solid lead sphere of mass m and radius r. The mass of the balloon is negligible as compared to the mass of the lead sphere. The system is buoyantly neutral, floating just below the surface in a pool of water.

The balloon is given a light downwards push. Describe the motion of the system qualitatively, and as quantitatively as possible.

Hint: A spherical object of radius r moving without turbulence and with a velocity v through a fluid of viscosity η experiences a retarding force given by,

$$F_v = 6\pi r\eta v \ .$$

Water has a viscosity of $\eta = 1.0 \times 10^{-3}$ PaS and a density of $\rho_w = 1000$ kg/m^3. Lead has a density of $\rho_l = 11.3 \times 10^3$ kg/m^3.

Link to Solutions