Canadian Association of Physicists
1998 Prize Exam

This is a three hour exam. National ranking and prizes will be based on a student's performance on both sections A and B of the exam. However, performance on the multiple choice questions in part A will be used to determine whose written work in part B will be marked for prize consideration by the National Committee. The questions in part B of the exam have a range of difficulty. Please be careful to gather as many of the easier marks as possible before venturing into more difficult territory. In some cases an answer to part (a) of a question is needed for part (b). Should you not be able to solve part (a), assume a likely solution and attempt the rest of the question anyway. No student is expected to completely finish this exam and parts of each question are very challenging.

Non-programmable calculators may be used. Please be very careful to answer the multiple choice questions on the answer card/sheet provided to you and, most importantly, write your solutions to the three written problems on separate sheets as they will be marked by different people in different parts of Canada. Good luck.

Data

Speed of light	$c = 3.00 \times 10^8$ m/s
Gravitational constant	$G = 6.67 \times 10^{-11}$ Nm^2/kg^2
Radius of Earth	$R_E = 6.38 \times 10^6$ m
Radius of Moon	$R_M = 1.70 \times 10^6$ m
Mass of Earth	$M_E = 5.98 \times 10^{24}$ kg
Mass of Moon	$M_M = 7.35 \times 10^{22}$ kg
Mass of Sun	$M_S = 1.99 \times 10^{30}$ kg
Radius of Moon's orbit	$R_{EM} = 3.84 \times 10^8$ m
Radius of Earth's orbit	$R_{ES} = 1.50 \times 10^{11}$ m
Acceleration due to gravity	$g = 9.81$ m/s^2
Fundamental charge	$e = 1.60 \times 10^{-19}$ C
Mass of electron	$m_e = 9.11 \times 10^{-31}$ kg
Mass of proton	$m_p = 1.673 \times 10^{-27}$ kg
Mass of neutron	$m_n = 1.675 \times 10^{-27}$ kg
Planck's constant	$h = 6.63 \times 10^{-34}$ Js
Coulomb's constant	$1/4\pi\epsilon_o = 8.99 \times 10^9$ Nm^2/C^2
Permeability of free space	$\mu_o = 4\pi \times 10^{-7}$ N/A^2
Speed of sound in air	$v_s = 340$ m/s
Density of air	$\rho = 1.2$ kg/m^3
Boltzmann constant	$k = 1.38 \times 10^{-23}$ J/K
Absolute zero	$T = 0$ K, $T = -273°$C
Energy Conversion	1 cal=4.184 J
Avogadro's number	$N_A = 6.02 \times 10^{23}$ mol^{-1}
Combustion Energy of Sugar	15 MJ/kg

Useful Relation:

For $x \ll 1$, $(1+x)^{-n} \approx (1-nx)$.

Part A: Multiple Choice

Question 1

What mechanical power P is required to move an object of mass m against an opposing force F at a constant velocity v?

(a) $P = Fv$

(b) $P = mv^2/2$

(c) $P = mv/F$

(d) $P = F^2/mv$

Question 2

A rocket generates a thrust force by ejecting hot gases from an engine. If it takes 1 ms to combust 1 kg of fuel, ejecting it at a speed of 1000 m/s, what thrust is generated?

(a) 1000 N

(b) 10,000 N

(c) 100,000 N

(d) 1,000,000 N

Question 3

A rubber band (two parallel strands of elastic material) has a spring constant of 10 N/m. If the band is cut in one place such that it now forms a single long strand of elastic material, what is its new spring constant?

(a) 20 N/m

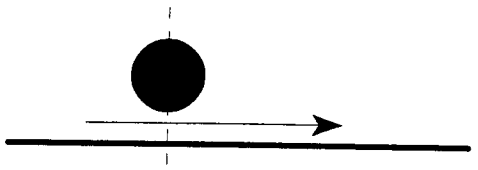
(b) 40 N/m

(c) 5 N/m

(d) 2.5 N/m

Question 4

A ball rests upon a flat piece of paper on a table top. The paper is pulled quickly and horizontally to the right as shown. Relative to the its initial position with respect to the table, the ball initially;



(a) moves to the right.

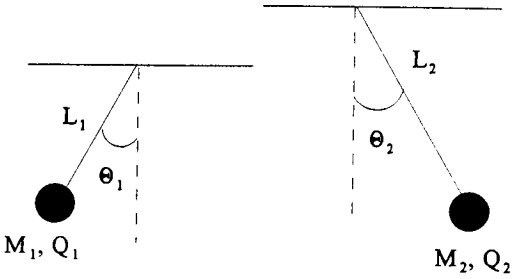
(b) moves to the left.

(c) remains stationary.

(d) its motion depends on the speed at which the paper was removed.

Question 5

Two small spheres with masses m_1 and m_2 hang on weightless, insulating threads with lenghts l_1 and l_2. The two spheres carry a charge of q_1 and q_2 respectively. The spheres hang such that they are level with one another and the threads are inclined to the vertical at angles θ_1 and θ_2. Which of the following conditions is required if $\theta_1 = \theta_2$.



(a) $m_1 = m_2$

(b) $\mid q_1 \mid = \mid q_2 \mid$

(c) $l_1 = l_2$

(d) none of the above

Question 6

An astronaut aboard the space shuttle, which is in orbit about the earth, is said to be weightless because;

(a) there is no force of gravity on her.

(b) a radially outwards force acts on her which cancels the force of gravity.

(c) she is falling in the earth's gravitational field.

(d) she is at such a position between the earth and the moon that their forces of gravity cancel each other.

Question 7

Beats are heard when the A strings of two violins are played. The beat frequency decreases as the tension in the A string of violin 1 is slowly increased. Which of the following statements is correct?

(a) The fundamental frequency of the A string in violin 1 is less than that for violin 2.

(b) The fundamental frequency of the A string in violin 1 is greater than that for violin 2.

(c) The fundamental frequency of the A string in violin 1 may be greater or less than that for violin 2 depending on the linear mass densities of the two strings.

(d) The fundamental frequency of the A string in violin 1 may be greater or less than that for violin 2 depending on the temperature.

Question 8

A proton (mass m_p, charge $+e$) and a deuteron (mass $2m_p$, charge $+e$) are initially at rest and are accelerated through the same potential difference ΔV. Which of the following is true concerning the final properties of the two particles?

(a) They have the same speed.

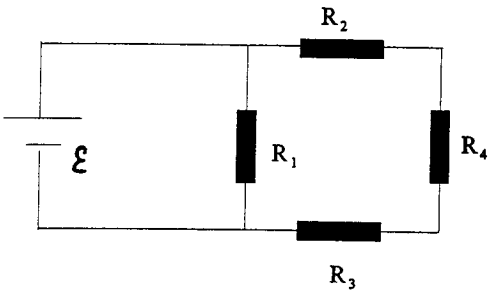
(b) They have the same momentum.

(c) They have the same kinetic energy.

(d) They have been subjected to the same impulse.

Question 9

An ideal battery is connected to a circuit of four resistors as shown. If the resistance of R_4 is increased, then:



(a) The current through R_1 decreases.

(b) The potential difference across R_1 increases.

(c) The potential difference across R_4 increases.

(d) The potential difference across R_2 decreases.

Question 10

A ray of light is incident upon an air/water interface (it passes from air into water) at an angle of 45°. Which of

(1) wavelength

(2) frequency

(3) speed of propagation

(4) direction of propagation

change as the light enters the water?

(a) 1,3 only

(b) 3,4 only

(c) 1,2,4 only

(d) 1,3,4 only

Question 11

Three freight cars, each of mass m, are being pulled by an engine exerting a force F on the lead car. What is the force exerted on the trailing car by the middle one?

(a) F

(b) $F/2$

(c) $F/3$

(d) $2F/3$

Question 12

The lowest frequency to resonate in a pipe, closed at one end and open at the other, is 300 Hz. Which of the following frequencies will not resonate in this particular pipe?

(a) 900 Hz

(b) 2,100 Hz

(c) 3,300 Hz

(d) 3,600 Hz

Question 13

A long metal bar, 30.0 cm in length , is aligned along a north-south line and moves eastwards at a speed of 10.0 m/s. A uniform magnetic field of 4.00 T points downwards. If the south end of the bar has a potential of 0 V, the induced potential at the north end of the bar is,

(a) +12 V

(b) -12 V

(c) 0 V

(d) cannot be determined since there is no closed circuit.

Question 14

A proton is moving in a uniform electric field, E, of unknown magnitude and direction. There is a uniform magnetic field of 0.01 T pointing in the y-direction. The proton moves in the x- direction at a constant velocity of 10 km/s. All directions are with respect to the usual right handed x-y-z coordinate system. Which of the following is correct?

(a) $E = 100$ V/m, in the z-direction.

(b) $E = 100$ V/m, in the negative z-direction.

(c) $E = 100$ V/m, in the x-direction.

(d) $E = 100$ V/m, in the negative y-direction.

Question 15

A aluminum block has a cavity within it which is completely closed. The block is placed in a region permeated by a uniform electric field which is directed upwards. Which of the following is a correct statement describing conditions in the interior of the block's cavity?

(a) The electric field in the cavity is directed upwards.

(b) The electric field in the cavity is directed downwards.

(c) There is no electric field in the cavity.

(d) The electric field in the cavity is of varying magnitude and is zero at the exact center.

Question 16

Two copper spheres, A and B, are identical in all respects but A carries a charge of -3 μC whereas B is charged to $+1$ μC. The two are brought together until they touch and then separated so that they are nearby but otherwise insulated. Which of the following is true concerning the electrostatic force F between the two spheres?

(a) $F = 0$ as one of the spheres is uncharged.

(b) $F = 0$ as both of the spheres are uncharged.

(c) F is attractive.

(d) F is repulsive.

Question 17

A real battery has an emf ϵ and an internal resistance r. A variable resistor R is connected across the terminals of the battery. A current I is drawn from the battery and the potential difference across the terminals of the battery is V. If R is slowly decreased to zero, which of the following best describes I, and V?

(a) I decreases to zero; V approaches ϵ.

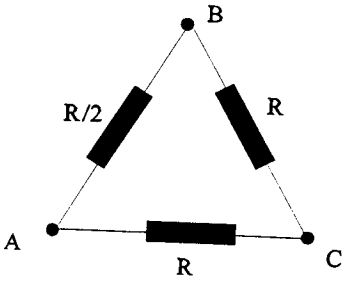
(b) I approaches an infinite value; V decreases to zero.

(c) I approaches ϵ/r; V approaches ϵ.

(d) I approaches ϵ/r; V decreases to zero.

Question 18

Three resistors are connected together as shown. A current I flows into A and out of B. What is the power dissipated by the resistor $R/2$?



(a) $8I^2R/25$

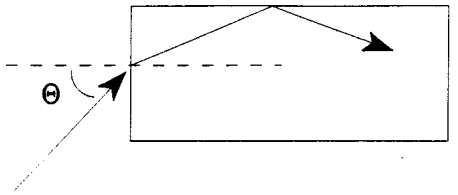
(b) I^2R

(c) $16I^2R/5$

(d) $25I^2R/32$

Question 19

Light enters one end of an optical fibre as shown. The uniform index of refraction of the cylindrical fibre is $n = 1.4$. What is the maximum angle θ that the incident light may have so that the light ray is totally internally reflected within the fibre. Take the index of refraction of air to be unity; all results shown to two significant digits.



(a) 78°
(b) 46°
(c) 44°
(d) 12°

Question 20

Two balls of the same mass are dropped from the same height onto the floor. The first ball bounces upwards from the floor elastically. The second ball sticks to the floor. The first applies an impulse to the floor of I_1 and the second applies an impulse I_2. The two impulses obey,

(a) $I_2 = 2I_1$.
(b) $I_2 = I_1/2$.
(c) $I_2 = 4I_1$.
(d) $I_2 = I_1/4$.

Question 21

A glass of water has an ice cube floating in it. The level of the water is marked on the glass with a grease-pencil. Once the ice cube has completely melted, and assuming that there has been no evaporation, the water level, relative to the initial marking, has;

(a) gone up.
(b) gone down.
(c) remained the same.
(d) cannot say based on the above information.

Question 22

A bit of mud stuck to a bicycle's front wheel detaches and is flung horizontally forward when it is at the top of the wheel's rotation (directly above the wheel's hub). The bicycle is moving forward at a speed v and the bicycle is rolling along without slipping. At the instant that the bit of mud hits the ground, how far in front of the bicycle's front wheel hub is it? (Ignore air resistance)

(a) $\sqrt{2rv^2/g}$
(b) $\sqrt{4rv^2/g}$
(c) $\sqrt{8rv^2/g}$
(d) $\sqrt{16rv^2/g}$

Question 23

An exploratory rocket of mass m is in orbit about the sun at a radius of $R_{ES}/10$ (one tenth of the radius of the earth's orbit about the sun). To exit this orbit, it fires its engine over a short period of time. This quickly doubles the velocity of the rocket while halving its mass (due to fuel consumption). Immediately after the burn, what is the kinetic energy of the rocket?

(a) $10GM_Sm/R_{ES}$
(b) $5GM_Sm/R_{ES}$
(c) $20GM_Sm/R_{ES}$
(d) $GM_Sm/2R_{ES}$

Question 24

A light bulb shines light along the x-axis and through two parallel ideal polarizing filters, one with a fixed polarizing axis, and one with an axis that rotates about the x-axis and in the yz-plane. Looking towards the light bulb through the combined filtering system, you see;

(a) A bulb almost disappearing twice per revolution, and reaching a maximum intensity twice per revolution. The maximum brightness is not as bright as looking at an unfiltered bulb.
(b) As in (a) but once per revolution.
(c) As in (a) but with a maximum brightness as bright as an unfiltered bulb.
(d) As in (a) but with a bulb that disappears completely.

Question 25

Two identical metallic spheres are both charged to $+Q$. The spheres are fixed in place, with their centers $2l$ apart from one another. They are joined by a light, in-flexible rod. Sliding back and forth on the rod (ignore friction) is a small disk of mass m charged to q. The disk is in equilibrium when it half way between the spheres. If the disk is displaced by a small amount from its equilibrium point, it will oscillate with a period of:

(a) $2\pi\sqrt{\pi\epsilon_o ml^3/Qq}$
(b) $2\pi\sqrt{Qq/\pi\epsilon_o ml^3}$
(c) $2\pi\sqrt{4\pi\epsilon_o ml^3/Qq}$
(d) $2\pi\sqrt{4\pi\epsilon_o ml^2/Qq}$

Part B

Question 1

A ball of mass $m = 1.0$ kg is dropped from a height $h_1 = 1.0$ m above a hard table top. It rebounds to a height $h_1' < h_1$. The coefficient of restitution between the ball and

the table is $\epsilon = \sqrt{h_1'/h_1} = 0.7$. The radius of the ball is $r = 0.5$ cm.

(a) What is the ratio of the ball's kinetic energy just after it has bounced to that just before it bounces?

(b) If the duration of the collision between the ball and the table top is $\Delta t = 0.01$ s, what is the average force exerted on the ball by the table?

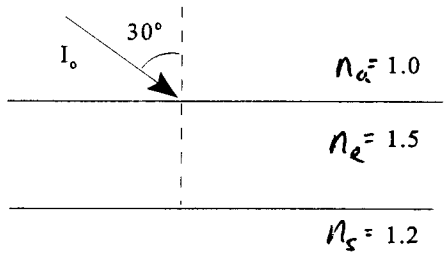
(c) From what height, h_2, above the table top and directly above the first ball, must a second ball be dropped so that it collides with the first ball immediately after the first ball has finished its bounce from the table top. The second ball is identical to the first but with a mass of 100 g.

(d) If the coefficient of restitution between the two balls is 1.0, find the rebound height, h_2', of the second ball above the table top.

Question 2

Laminar optical devices are important in the semiconducting electronics industry. It is important to know how light interacts with such an interface and so let's consider the interaction of light with a simple laminar structure.

Suppose that a beam of light of intensity I_o shines upon a laminar interface with a small angle of incidence $\theta_i = 30°$. The interface is a laminar *sandwich* made of two layers. There is a thick substrate layer , with an index of refraction $n_s = 1.2$, and a thin mid-layer, with an index of refraction $n_l = 1.5$. The index of refraction of air, n_a may be taken to be unity. The thickness of the mid-layer is large enough so that interference effects are not an issue.



(a) Find the angle θ_l that light refracted through the air/mid-layer interface makes with the normal. Hence, find the angle of incidence for light impinging upon the substrate. Finally, find the refracted angle, θ_s, for light passing through the mid-layer/substrate interface.

Note: For unpolarized light incident upon an n_1/n_2 interface with an incident angle θ_i, there will be a reflected ray and a refracted ray (at an angle of θ_r to the normal). The fraction of the incident light reflected is given by,

$$R = \frac{1}{2}\left(\frac{\tan^2(\theta_r - \theta_i)}{\tan^2(\theta_r + \theta_i)} + \frac{\sin^2(\theta_r - \theta_i)}{\sin^2(\theta_r + \theta_i)}\right).$$

The fraction of light transmitted is $T = 1 - R$.

(b) Using your angles from part (a), calculate values for R and T for reflection and transmission at the air/mid-layer, mid-layer/substrate, and mid-layer/air interfaces.

(c) Find an expression for the intensity of the light which is reflected from the laminar interface. Evaluate your expression to as many significant figures as possible.

Question 3

A hummingbird has a mass of $m = 10$ g. It hovers in much the same manner as a helicopter except that instead of rotating, its wings sweep back and forth, developing lift on both parts of the stroke. The vertical, cross sectional area that the wings sweep through is $A = 0.01$ m^2.

(a) Estimate the mechanical power output P the hummingbird needs to generate for hovering.

(b) Estimate the mass of sugar the hummingbird needs to ingest (as nectar) in order to hover for one hour.

Note: This question requires you to make assumptions and to estimate physical quantities. Marks are awarded for the quality of these assumptions and estimations so please explain them clearly. You should, however, attempt the question using your assumptions whatever the quality of your estimations.

Link to Solutions