

CAP High School Prize Exam

April 8 2005

9:00 – 12:00

Competitor's Information Sheet

The following information will be used to inform competitors and schools of the exam results, to determine eligibility for some subsequent competitions, and for statistical purposes. Only the marking code, to be assigned by the local examination committee, will be used to identify papers for marking.

Marking Code:

This box must be left empty.

PLEASE PRINT CLEARLY IN BLOCK LETTERS.

Family Name: _____ Given Name: _____

Home Address: _____

_____ Postal Code: _____

Telephone: () _____ E-mail: _____

School: _____ Grade: _____

Physics Teacher: _____

Date of Birth: _____ Sex: _____

Citizenship: _____

For how many years have you studied in a Canadian school? _____

Would you prefer further correspondence in French or English? _____

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Canadian Association of Physicists
Canadian Chemistry and Physics Olympiads

Canadian Association of Physicists 2005 Prize Exam

This is a three-hour exam. National ranking and prizes will be based on a student's performance on both sections A and B of the exam. Performance on the multiple-choice questions in part A will be used to determine whose written work in part B will be marked for prize consideration by the CAP Exam National Committee. The questions in part B have a range of difficulty. Do be careful to gather as many of the easier marks as possible before venturing into more difficult territory. If an answer to part (a) of a question is needed for part (b), and you are not able to solve part (a), assume a likely solution and attempt the rest of the question anyway. No student is expected to complete this exam and parts of each problem may be very challenging.

Non-programmable calculators may be used. Please be careful to answer the multiple-choice questions **on the answer card/sheet** provided; most importantly, write your solutions to the three long problems on **three separate** sheets as they will be marked by people in different parts of Canada. Good luck.

Data

Speed of light	$c = 3.00 \times 10^8 \text{ m/s}$
Gravitational constant	$G = 6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$
Radius of Earth	$R_E = 6.37 \times 10^3 \text{ km}$
Mass of Earth	$M_E = 6.0 \times 10^{24} \text{ kg}$
Radius of Earth's orbit	$R_{ES} = 1.50 \times 10^8 \text{ km}$
Acceleration due to gravity	$g = 9.80 \text{ m/s}^2$
Fundamental charge	$e = 1.60 \times 10^{-19} \text{ C}$
Mass of electron	$m_e = 9.11 \times 10^{-31} \text{ kg}$
Mass of proton	$m_p = 1.673 \times 10^{-27} \text{ kg}$
Planck's constant	$h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s}$
Coulomb's constant	$1/4\pi\epsilon_0 = 8.99 \times 10^9 \text{ J}\cdot\text{m}/\text{C}^2$

Part A: Multiple Choice

Question 1

A child throws a ball toward the front end of an approaching train. The collision between the ball and the train is perfectly elastic. Let v be the speed of the ball with respect to the train and V its speed with respect to the ground. If the labels "i" and "f" refer to those speeds just before and just after, respectively, the ball hits the train, then

- (a) $v_i = v_f$ and $V_i < V_f$; (b) $v_i < v_f$ and $V_i < V_f$;
 (c) $v_i > v_f$ and $V_i < V_f$; (d) $v_i = v_f$ and $V_i > V_f$.

Question 2

A physics textbook of mass m rests flat on a horizontal table of mass M placed on the ground. Let $N_{a \rightarrow b}$ be the contact force exerted by body "a" on body "b". According to Newton's 3rd Law, which of the following is an action-reaction pair of forces?

- (a) mg and $N_{\text{table} \rightarrow \text{book}}$;
 (b) $(m + M)g$ and $N_{\text{table} \rightarrow \text{book}}$;
 (c) $N_{\text{ground} \rightarrow \text{table}}$ and $Mg + N_{\text{book} \rightarrow \text{table}}$;
 (d) $N_{\text{ground} \rightarrow \text{table}}$ and $N_{\text{table} \rightarrow \text{ground}}$.

Question 3

At some time t , two identical balls, A and B, are set rolling without slipping at the same speed from one end of two tracks which are identical with the same horizontal length, except that track B has a dip in the path of ball B, as shown in the figure. The straight portions of the tracks are horizontal. Gravity is uniform throughout. Which ball reaches the other end first?



- (a) Ball A;
 (b) neither, since both arrive at the same time;
 (c) ball B;
 (d) ball A, but only if the dip is deep enough.

Question 4

To a good approximation, Earth and Jupiter move around the Sun in circular orbits of $1.49 \times 10^8 \text{ km}$ and $7.79 \times 10^9 \text{ km}$ radius, respectively. What is the maximum error that can arise in the prediction of solar eclipse times (as observed from Earth) on Jupiter caused by one of its moons if one fails to take into account the variation of the relative position of the two planets?

- (a) $2.6 \times 10^3 \text{ s}$; (b) $3.1 \times 10^3 \text{ s}$;
 (c) 5.0×10^2 ; (d) $9.9 \times 10^2 \text{ s}$.

Question 5

Experiment shows that two perfectly neutral parallel metal plates separated by a small distance d , attract each other via a very weak force, known as the Casimir force. The force per unit area of the plates, \mathcal{F} , depends only on the Planck constant h , on the speed of light c , and on d . Which of the following has the best chance of being correct for \mathcal{F} ?

- (a) $\mathcal{F} = hc/d^2$; (b) $\mathcal{F} = hc/d^4$;
 (c) $\mathcal{F} = hd^2/c$; (d) $\mathcal{F} = d^4/hc$.

Question 6

A simple pendulum of length L with a bob of mass m is taken into Earth orbit on the International Space Station. Its frequency of oscillation with respect to that on the ground is

- (a) greater; (b) smaller but non-zero;
 (c) the same; (d) zero.

Question 7

When you turn on a battery-operated portable music-playing device, how long must you leave it on for electrons leaving the negative terminal of the battery to reach the positive terminal if their path lies within good conductors?

- (a) A few milliseconds; (b) a few tenths of a second;
 (c) a few microseconds; (d) a few minutes.

Question 8

Circuit A is made of resistors connected in series to a battery; circuit B is made of resistors connected in parallel to a battery. Let P be the power drawn from the batteries. As the number of resistors in each circuit is increased,

- (a) P_A increases and P_B decreases;
- (b) both P_A and P_B increase;
- (c) P_A decreases and P_B increases;
- (d) P_A and P_B remain the same.

Question 9

According to a simplified but still useful model, the drag force due to air resistance on a moving car goes like the square of the car's speed v . Suppose that the maximum speed of a car is limited only by this drag force. If the power of the car's engine were increased by 50%, the top speed of the car would increase by about

- (a) 50%;
- (b) 15%;
- (c) 22%;
- (d) 30%.

Question 10

A projectile is launched with an initial velocity \mathbf{v}_i , with v_{ix} and v_{iy} the horizontal and vertical velocity components, respectively. When is there a point on its trajectory after launch where its velocity is perpendicular to its acceleration?

- (a) Always;
- (b) only if $v_{ix} \neq 0$ and v_{iy} points upward;
- (c) only if $v_{ix} \neq 0$;
- (d) always, except if $v_{ix} = 0$.

Question 11

Three charged conducting metal spheres, of radius R_1 , R_2 , and R_3 , are connected together by wires. Let $R_1 < R_2 < R_3$. At equilibrium, which of the following sets of relations involving the electric field strength E generated by a sphere, its potential V , and its charge Q , must hold between the spheres?

- (a) $V_1 = V_2 = V_3$, $E_1 < E_2 < E_3$, $Q_1 < Q_2 < Q_3$;
- (b) $V_1 = V_2 = V_3$, $E_1 > E_2 > E_3$, $Q_1 < Q_2 < Q_3$;
- (c) $V_1 < V_2 < V_3$, $E_1 < E_2 < E_3$, $Q_1 = Q_2 = Q_3$;
- (d) $V_1 > V_2 > V_3$, $E_1 < E_2 < E_3$, $Q_1 > Q_2 > Q_3$;

Question 12

A ball of mass m attached to an inextensible string of length R in swung around a vertical circle just fast enough so that the string is always fully stretched. Let ΔT denote the difference between the tension in the string at the bottom and at the top of the circle, v_b and v_t the speed of the ball at the bottom and at the top, respectively. Then, taking "dependence" to be with respect to a set of independent variables,

- (a) ΔT is independent of R , v_b and v_t ;
- (b) ΔT is independent of R , but depends on $v_b^2 - v_t^2$;
- (c) ΔT depends on R , but on neither v_b nor v_t ;
- (d) ΔT depends on R and $v_b^2 - v_t^2$.

Question 13

An object of mass m hangs motionless from a vertical spring. When the object is pulled down to a new rest position, the total mechanical energy of the system

- (a) increases; (b) remains the same;
- (c) decreases; (d) may increase or decrease depending on the new position.

Question 14

As more and more negative electric charge is being brought to a conducting sphere, inside the sphere

- (a) the electric field and potential increase;
- (b) the electric field stays constant and the potential increases;
- (c) the electric field stays constant and the potential decreases;
- (d) the electric field increases and the potential decreases.

Question 15

A static magnetic field of about 0.01 T in strength can erase data on the magnetic strip of a credit card. What would be roughly the minimum diameter of a long straight wire carrying a 100 A current for which your card would be safe no matter how close you take it to the wire?

- (a) 0.2 mm; (b) 1 mm;
- (c) 2 mm; (d) 4 mm.

Question 16

A string of length L is composed of two segments of equal length. One segment has linear mass density μ_1 and the other $\mu_2 \neq \mu_1$. One segment is tied to a wall, and the string is stretched by a force, applied to the other segment, which is much greater than the total weight of the string. If T_i is the tension in the i^{th} segment, and v_i the speed of a transverse wave propagating along that segment,

- (a) $v_1 = v_2$ and $T_1 = T_2$;
- (b) $v_1 \neq v_2$ and $T_1 = T_2$;
- (c) $v_1 = v_2$ and $T_1 \neq T_2$;
- (d) $v_1 \neq v_2$ and $T_1 \neq T_2$.

Question 17

A perfectly straight portion of a uniform rope has mass M and length L . At end A of the segment, the tension in the rope is T_A ; at end B it is $T_B > T_A$. The tension in the rope at a distance $L/5$ from end A is

- (a) $T_B - T_A$;
- (b) $(T_A + T_B)/5$;
- (c) $(4T_A + T_B)/5$;
- (d) $(T_A - T_B)/5$.

Question 18

Two spheres are identical except that sphere A is white whereas sphere B is black. After they have been in thermal contact long enough with each other and their surroundings, in the visible range,

- (a) A radiates less than B;
- (b) both emit the same amount of radiation;
- (c) A radiates more than B;
- (d) A radiates more than B only if its temperature is high enough.

Question 19

An aircraft bound for Vancouver and coming from Montréal is flying due west. Its body and wings are covered in aluminium. At some point on its flight path, the Earth's magnetic field points north and downward. The point on the plane's exterior which is then at the highest potential is

- (a) the nose (front); (b) the tail (back);
(c) the tip of the right wing; (d) the tip of the left wing.

Question 20

You are on the shore of a canal of uniform width d and want to reach a point a distance $L > d$ away *along* the other shore as quickly as possible. To achieve this, you first run along the shore at constant speed v_1 , then jump in the canal and swim directly toward your target at constant speed $v_2 < v_1$, both being your maximum speeds. The water in the canal is motionless. The angle of your trajectory in the water with respect to the shore must obey:

- (a) $\cos \theta = v_1/v_2$; (b) $\cos \theta = \sqrt{v_1^2/v_2^2 - 1}$;
(c) $\cos \theta = 1 - d/L$; (d) $\cos \theta = v_2/v_1$.

Question 21

Exactly half of a rectangular conducting loop lies in a uniform magnetic field perpendicular to the plane of the loop. At some point in time, the magnitude of the magnetic field starts rapidly decreasing. While this is happening, which of the following statements most accurately describes the effect on the loop?

- (a) The loop is pulled into the magnetic field;
(b) the loop is pushed out of the magnetic field;
(c) the loop starts rotating;
(d) the behaviour of the loop cannot be determined unless the direction of the magnetic field is completely specified.

Question 22

Two otherwise identical spaceships have different solar sails: sail A is a perfect reflector, sail B is a perfect absorber. Each starts at the same distance from the Sun and travels radially outward. Let Δp_A and Δp_B be the momentum gained by the ships after travelling equal distances. Then

- (a) $\Delta p_A = \Delta p_B$; (b) $\Delta p_A > \Delta p_B$;
(c) $\Delta p_A < \Delta p_B$; (d) $\Delta p_A = \Delta p_B = 0$.

Question 23

If there were only one transmitter, and you were separated from this transmitter by the many tall buildings to be found in downtown Toronto, spaced about 30 m apart on average, with which of the following would you be most likely to experience dead spots (places with very poor or no reception)?

- (a) AM radio stations (frequency 1 MHz);
(b) FM radio stations (frequency 100 MHz);
(c) cell phones (frequency 1000 MHz);
(d) all of the previous equally.

Question 24

A horizontal cathode ray tube (CRT) is set so that its electron beam produces a spot of light at the centre of the screen when no external electromagnetic field is present. When you look straight at the screen, however, you discover that the spot, instead of being at the centre as it should, is shifted a bit to the right. Suspecting what the cause of this deflection may be, you rotate the CRT by 180° around its vertical axis. Facing the screen, you find that the spot of light is still shifted to right of centre by the same distance as before. You conclude that the CRT is immersed in

- (a) an electric field directed horizontally to the left with respect to the screen's initial position;
(b) an electric field directed horizontally to the right with respect to the screen's initial position;
(c) a magnetic field directed vertically upward with respect to the screen's initial position;
(d) a magnetic field directed vertically downward with respect to the screen's initial position;

Question 25

Light rays from a very distant source travel along the $+x$ direction. Two identical thin lenses with focal length $f > 0$ and their optical axis along x , sit, one at $x = 0$, and the other at $x = d < f$. Where do the rays focus?

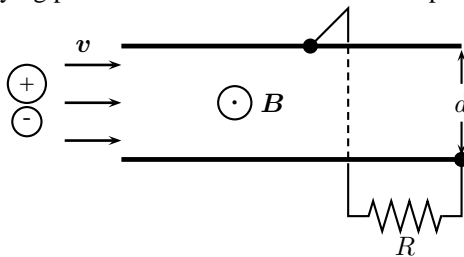
- (a) $d + \frac{f(f-d)}{2f-d}$; (b) $d - \frac{f(f-d)}{d}$;
(c) $d + \frac{f(f-d)}{2(f+d)}$; (d) $d + \frac{f^2}{2f-d}$.

* * *

Part B

Problem 1

A magnetohydrodynamic (MHD) generator is a device that converts part of the kinetic energy of a streaming hot gas into electrical energy. At its operating temperature of between 2000 and 3000 K, the gas is readily ionised. As schematically shown in the figure, the ions and electrons enter a region between two electrodes (here, parallel conducting plates) of area A separated by a gap d in which a uniform and constant magnetic field B has been set up, pointing straight out of the page. Their initial velocity v is parallel to the plates. A resistor R is connected to the plates. The magnetic field is strong enough that many charged particles will hit the electrodes and charge them before they can exit. This creates a time-varying potential difference V between the plates.



- (a) On a diagram, display all the forces acting on a positive ion at some generic position between the plates. Use your diagram to show that the ion experiences a braking force as soon as its velocity has a component transverse to the plates.
- (b) Then, as a crude approximation, assume that any electric field E arising from the process is uniform everywhere between the plates. Assume also that the transverse component of the velocity of the charges remains everywhere much smaller than its longitudinal component, and that the latter therefore is pretty uniform. The gas obeys a generalised Ohm's law, according to which $\mathbf{J} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B})$, where \mathbf{J} is the transverse current density between the plates and σ is the conductivity of the gas.
- Obtain an expression for the current I flowing through the resistor in terms of v , B , and R .
- (c) Derive an expression for the output voltage V that does not explicitly contain R .

Problem 2

A 1000 kg satellite is orbiting the Earth at an altitude of 400 km. It receives electrical power from a solar panel of area $A = 10 \text{ m}^2$. At this altitude, Earth's atmosphere is very tenuous, with a density $\rho \approx 10^{-11} \text{ kg/m}^3$. Nevertheless, over time, the friction force generated by collisions of the molecules and the panel might cause the satellite to lose altitude.

- (a) Assume that in such collisions, the molecules become embedded in the solar panel. If the satellite is moving at speed v , find an expression for the maximum retarding force on the solar panel in terms of ρ and of the radius of the satellite's orbit. Make any other reasonable assumption.
- (b) Estimate how much altitude the satellite might lose over one week because of this friction. If you make assumptions, do not forget to justify them briefly.
- (c) Somebody claims that as the satellite loses altitude it also loses speed because of the friction. Comment briefly.

Problem 3

In a mood for some physics as you look at a sailboat on a lake, you wonder about its stability when a strong wind blows from the side. The sailboat leans over in the wind and the question is whether its keel can prevent it from being blown over completely.

Analyze the stability of the sailboat using the following over-simplified model.

Consider the hull of the sailboat as an airtight hollow cylinder. On top of this cylinder and perpendicular to it sits a mast carrying a square sail, assumed to be always parallel to the length of the hull. Attached to the bottom of the hull is a keel in the shape of a square always parallel to the sail. A heavy lead weight forms the bottom of the keel under water. Other than the lead weight, consider mast, sail, hull, and keel to be weightless. Somewhat unrealistically (it would make tacking difficult), you can also assume that the sail extends all the way from the bottom to the top of the mast.

- (a) Establish a relationship between wind speed and the angle θ the mast will tilt away from its initial upright position when a wind with speed v blows perpendicular (initially) to the sail. Other assumptions may be made so long as they are explicitly stated and, as much as possible, justified.

In this simplified model, is this boat stable in all wind speeds? Since you may not be able to find a general solution for the angle θ as a function of v , you are welcome to obtain solutions valid only for small θ or for large θ . The expansion $(1+x)^n \approx 1+nx+\dots$ for $x \ll 1$ may be useful.

- (b) The following data, loosely based on the *Catalina Capri* – 16, are given for a small sailboat. Area A of sail is 12 m^2 , mass M of lead weight 190 kg, depth d of keel below water level, technically known as draft, 0.75 m, height h of mast 6.6 m, specific gravity of lead, supposed to be the part at the bottom of the keel that acts as a stabiliser, 11.3. Also, air density is 1.2 kg/m^3 .

With these data, at what wind speed will θ be 30° ? 60° ?