CAP High School Prize Exam

12 April 2010
9:00 – 12:00

Competitor's Information Sheet

The following information will be used to inform competitors and schools of the exam results, to determine eligibility for some subsequent competitions, and for statistical purposes. Only the marking code, to be assigned by the local examination committee, will be used to identify papers for marking.

Marking Code:

This box must be left empty.

PLEASE PRINT CLEARLY IN BLOCK LETTERS.

Family Name:________________________________Given Name:_______________________

Home Address:____

____ Postal Code:______________

Telephone: (   )____________________ E-mail: __________________________________

School: ____ Grade:_____

Physics Teacher: ____

Date of Birth: ___________________________ Sex: Male ☐ Female ☐

Citizenship: ____ or

Immigration Status:____

For how many years have you studied in a Canadian school? ____________________________

Would you prefer the further correspondence in French or English? _______________________

Sponsored by:

Canadian Association of Physicists
Canadian Chemistry and Physics Olympiads
University of Toronto, Department of Physics

Canadian Association of Physicists
2010 Prize Exam

This is a three-hour exam. National ranking and prizes will be based on a student's performance on sections A and B of the exam. Performance on the questions in parts A will be used to determine whose written work in part B will be marked for prize consideration by the CAP Exam National Committee. Part A consists of twenty-two multiple-choice questions. The questions in part C span a range of difficulties, and may require graphing. Do be careful to gather as many of the easier marks as possible before venturing into more difficult territory. If an answer to part (a) of a question is needed for part (b), and you are not able to solve part (a), assume a likely solution and attempt the rest of the question anyway. No student is expected to complete this exam and parts of each problem may be very challenging.

Non-programmable calculators may be used. Please be careful to answer the multiple-choice questions **on the answer card/sheet** provided; most importantly, write your solutions to the three long problems on **three separate** sheets as they will be marked by people in different parts of Canada. Good luck.

Data

Speed of light $c = 3.00\text{x}10^8$ m/s
Gravitational constant $G = 6.67\text{x}10^{-11}$ N·m²/kg²
Acceleration due to gravity $g = 9.80$ m/s²
Density of fresh water $\rho = 1.00\text{x}10^3$ kg/m³
The normal atmospheric pressure $P_0 = 1.01\text{x}10^5$Pa
The specific heat of water $c = 4{,}186\text{x}10^3$J/(kg·K)
Fundamental charge $e = 1.60\text{x}10^{-19}$ C
Mass of electron $m_e = 9.11\text{x}10^{-31}$ kg
Mass of proton $m_p = 1.67\text{x } 10^{-27}$ kg
Planck's constant $h = 6.63\text{x}10^{-34}$ J·s
Coulomb's constant $1/(4\pi\varepsilon_0) = 8.99\text{x}10^9$ N·m²/C²
Boltzmann constant $k = 1.38\text{x}10^{-23}$ J/K

Part A: Multiple Choice

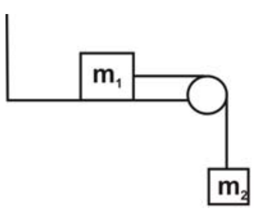
Question 1

A ball is dropped from a height h. It hits the ground and bounces back up with a momentum loss of 10% due to the impact. The maximum height it will reach is

a) 0.90 h; b) 0.81 h; c) 0.949 h; d) 0.3 h.

Question 2

A mass m_1 rests on a frictionless surface. It is attached to mass m_2 by a light string which passes over a massless, frictionless pulley, as shown in the figure. The system is released from rest. The initial acceleration a of m_2 is given by

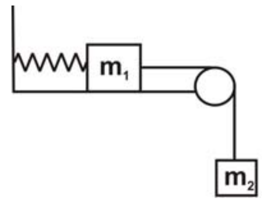


a) $a = g$; b) $a = \dfrac{m_2}{(m_1+m_2)} g$;

c) $a = (m_2 - m_1)g$; d) $a = \dfrac{m_1}{(m_1+m_2)} g$.

Question 3

The system in question 2 is attached to a spring of spring constant k which in turn is attached to a vertical wall, as shown to the right. The spring is neither stretched nor compressed. The system is released from rest. The mass m_2 falls and then oscillates about a mean displacement x from its original position before release given by

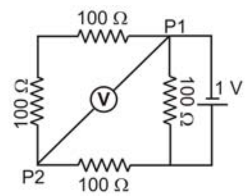


a) $x = \dfrac{m_2 g}{k}$; b) $x = \dfrac{m_1 g}{k}$;

c) $x = \dfrac{(m_1+m_2)g}{k}$; d) $x = \dfrac{k}{m_2 g}$.

Question 4

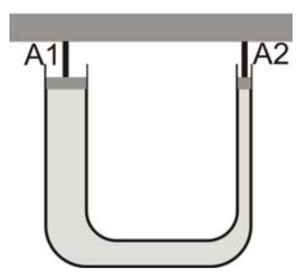
Four 100 ohm resistors and a 1 V battery are connected as shown in the circuit diagram. The potential difference between points P1 and P2 is given by



a) 0.250 V; b) 0.333 V; c) .500 V; d) 0.667 V.

Question 5

A heavy plate is held horizontal resting on two frictionless pistons, each of which is placed in the opening of an asymmetric U tube filled with an incompressible fluid, as shown in the figure. Once the restraints on the plate are released, then



a) piston A1 will rise and A2 will fall;
b) piston A2 will rise and A1 will fall;
c) the plate will remain level and not move;
d) the plate will remain level and both pistons will sink a similar small distance.

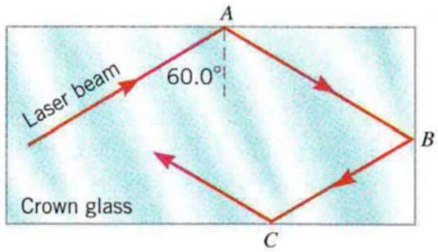
Question 6

Two identical glasses, one completely filled with water, and the other filled with vodka, are viewed directly from above. The refractive index of vodka is slightly greater than the refractive index of water. Which glass appears to have a greater depth of fluid?

a) the water glass;
b) the vodka glass;
c) neither glass because the apparent depth is the same as the real depth in both glasses;
d) neither glass, because the effect of the clear fluid is to reduce the apparent depth by the same factor in both glasses.

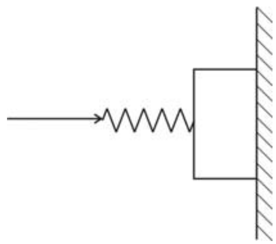
Question 7

The drawing shows a crown glass slab of refractive index 1.5 with a rectangular cross section. As illustrated, a laser beam strikes the upper surface at an angle of 60°. After reflecting from the upper surface, the beam reflects from the side and bottom surfaces. If the glass is surrounded by air, determine at which point part of the beam first exits the glass.



a) A; b) B; c) C; d) none of the above.

Question 8
A block of mass m is sandwiched between a rough vertical wall and a horizontally oriented spring with spring constant k. If the spring is compressed a distance x beyond its uncompressed length, then the minimum coefficient of static friction for which the block does not fall is



a) $\frac{mg}{kx}$; b) $\frac{kx}{mg}$; c) $\frac{kx}{mg+kx}$; d) $\frac{mg}{mg+kx}$.

Question 9
A light wave has a high enough frequency to ionize an atom. If the amplitude of the light wave is tripled, the chance of an atom being ionized by the light wave increases by a factor of

a) 1; b) $\sqrt{3}$; c) 3; d) 9.

Question 10
A 100-kg student eats a 200-calorie doughnut. To 'burn off the sugar high', he decides to climb the steps of a tall building. How high would he have to climb to expend an equivalent amount of work? (1 food calorie = 10^3 calories ~ 4200 J).

a) 273 m; b) 428 m; c) 857 m; d) 8400 m.

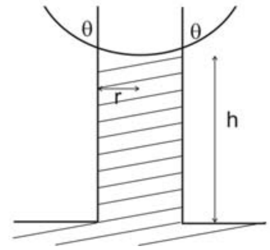
Question 11
In an insulated vessel, 250 g of ice at 0°C is added to 600 g of water at 18°C. The latent heat of fusion is ~ 3.33×10^5 J/kg. The amount of ice remaining when the system reaches thermal equilibrium is

a) 0 g; b) 114 g; c) 136 g; d) 45.2 g.

Question 12
A liquid will rise in a narrow tube (capillary action) if the adhesive forces between the molecules of the liquid and the molecules of the tube are greater than the cohesive forces between the molecules of the liquid. The height of a column of liquid of density ρ drawn up a tube of radius r is given by

$$h = \frac{2\gamma}{\rho g r}\cos\theta$$

where θ is the contact angle between the surface of the liquid and the tube, g is the acceleration due to gravity, and γ is a constant that depends on a number of parameters. The dimensions of γ are given by

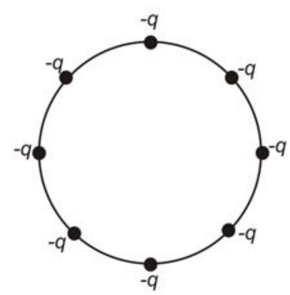


a) ML^{-2}; b) MLT^{-2}; c) $M^{-1}LT^{-2}$
d) MT^{-2}; e) none of the above.

Question 13
If the Earth's polar land-based glaciers melted, water would be redistributed on the planet, and the depression in the earth's crust made by the weight of the ice would rebound, as the land rises when the weight of the glaciers is removed (an effect known as Post-Glacial Rebound or Glacial Isostatic Adjustment). Following post-glacial rebound, the length of day would

a) increase;
b) decrease;
c) remain the same as the present length of day;
d) at first increase but then shortly return to the present length of day;
e) at first decrease but then shortly return to the present length of day.

Question 14
Eight identical negative charges $-q$ are arranged symmetrically on a circle of radius r, each equidistant from the next. Assume the point charges have zero potential at infinity. If the constant of proportionality in Coulomb's law is k, then the magnitude of the electric field and the electrostatic potential at the centre of the circle are given by

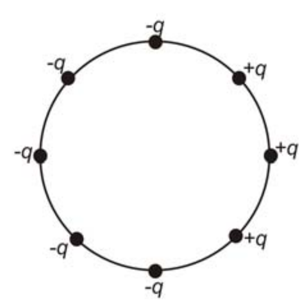


a) $E = \frac{8kq}{r^2}$, $V = \frac{-8kq}{r}$; b) $E = \frac{8kq}{(2\pi r)^2}$, $V = \frac{-8kq}{(2\pi r)}$;

c) $E = \frac{8kq}{r^2}$, $V = 0$; d) $E = 0$, $V = 0$;

e) $E = 0$, $V = \frac{-8kq}{r}$.

Question 15
Three of the negative charges in the previous questions are replaced with positive charges with the same magnitude, and are arranged as in the figure. The magnitude of the electric field and the electrostatic potential at the centre of the circle are given by



a) $E = \frac{2kq}{r^2}\left(1+2\sqrt{2}\right), V = 0$;

b) $E = \frac{2kq}{r^2}\left(1+\sqrt{2}\right),\ V = \frac{-2qk}{r}$;

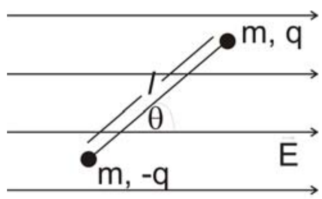
c) $E = \frac{2kq}{r^2}\left(1+2\sqrt{2}\right),\ V = \frac{-2qk}{r}$;

d) $E = 0,\ V = \frac{-2qk}{r}$.

e) None of the above.

Question 16

An electric dipole consists of two small particles. Each has mass m but opposite electric charge $\pm q$. They are separated by a light, non-conducting rod of length l. The dipole is placed in a uniform electric field $\vec{E}$ such that it forms an angle θ to the field, then is released. Its initial angular acceleration about its centre of mass is given by



a) $\frac{2qE\cos\theta}{ml}$; b) $\frac{2qE\sin\theta}{ml}$; c) $\frac{qEl\cos\theta}{m}$;

d) $\frac{qEl\sin\theta}{m}$; e) $\frac{qE\sin\theta}{ml^2}$.

Question 17

At time $t = 0$ a block of mass m moves with momentum p along a rough horizontal surface. If it comes to a complete stop in time t, then the coefficient of kinetic friction μ_k is given by

a) $\frac{p}{mgt}$; b) $\frac{pg}{mt}$; c) $\frac{ptg}{m}$; d) $\frac{pt}{mg}$;

e) None of the above.

Question 18

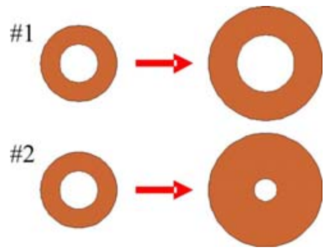
A fish swimming in water with constant velocity v_0 experiences a viscous drag force that is proportional to the square of its velocity (i.e. $F_{drag} \propto v^2$). If the fish doubles its velocity, then the thrust force F that it must generate to maintain this new velocity is related to the fish's original thrust force F_0 via

a) $F = 2F_0$; b) $F = \sqrt{2}F_0$; c) $F = 4F_0$; d) $F = F_0{}^2$;

e) None of the above.

Question 19

A solid object has a hole in it. Which of these illustrations more correctly shows how the size of the object and the hole change as the temperature increases?



a) illustration #1;

b) illustration #2;

c) The answer depends on the material of which the object is made;

d) The answer depends on how much the temperature increases;

e) Both c) and d) are correct.

Question 20

Two gear wheels of the same thickness, but with one twice the diameter of the other, are mounted on parallel light axles far enough apart not to mesh the teeth of the wheels. The larger wheel is spun with angular velocity Ω and the wheels are then moved together so they mesh. What is the subsequent angular velocity of the larger wheel?

a) $\frac{\Omega}{2}$; b) $\frac{\Omega}{5}$; c) $\frac{2\Omega}{5}$; d) $\frac{4\Omega}{5}$.

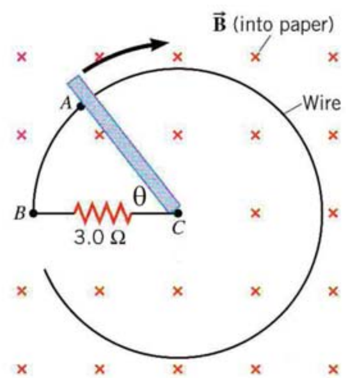
Question 21

For the meshed rotating gears of the previous question, what fraction of the original energy is lost?

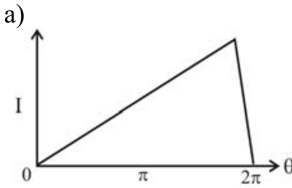
a) 1/4; b) 1/8; c) 1/5; d) 2/5.

Question 22

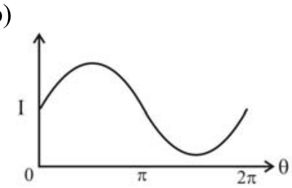
The drawing shows a copper wire (of negligible resistance) bent into a circular shape with a radius of r. The radial section BC is fixed in place, while the copper bar AC sweeps around at an angular speed of ω. The bar makes electrical contact with the wire at all times. The wire and the bar have negligible resistance. A uniform magnetic field exists everywhere, is perpendicular to the plane of the circle, and has a magnitude of B. Which of the following shows the magnitude of the current as a function of θ?



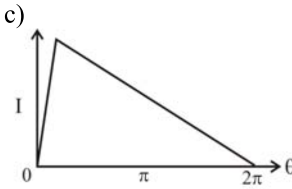
a)



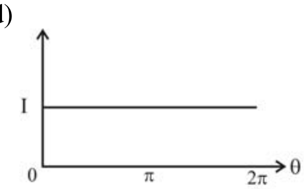
b)



c)



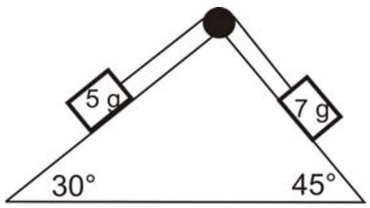
d)



Part B: Problems

Problem 1

A double inclined plane is in the form of a wedge of weight 10 g, held on the ground with its base horizontal and rough faces inclined at 30° and 45° respectively to the horizontal. A 5 g particle on the face inclined at 30° is connected by a mass less, flexible, inextensible string over a smooth mass less pulley at the ridge of the wedge to a 7 g particle on the other face. If the coefficient of friction between each of the particles and the wedge is 1/5, find the accelerations of the particles and the magnitude of the vertical force between the wedge and the ground.



Problem 2

Oceanographers have deployed an array of instruments off the west coast of Vancouver Island, connected together by cables carrying power and communication. The power is supplied through a single DC line. The return path is through the sea water. The grounds at each end of the line are large metal spheres.

Consider two copper spheres, radii *a*, buried in an infinite medium of resistivity ρ. Current flows out of one sphere and is collected by the second sphere. The current flow in vicinity of each sphere can be considered radially uniform as the distance between the spheres is large compared with *a*.

a) By considering the series resistance of many thin spherical shells around a sphere show graphically (or by direct integration) that much of the resistance between the spheres is due to material within a radius 2*a* of each sphere and thereby estimate the total resistance between the spheres.
b) If the current flow between the spheres is 1 A, calculate the charge in coulombs on each one.

Problem 3

Sir Ernest Rutherford, a very famous New Zealander who worked at McGill University in Montreal and the Cavendish Laboratory in Cambridge, won the Nobel Prize for his studies of the disintegration of nuclei. He established an important relationship for a nuclear reaction, in which a parent atom disintegrates into a daughter atom, which is called the Rutherford – Soddy law of radioactive disintegration. This law states that if N is the number of radioactive nuclei present at some instant, the number of nuclei ΔN that decay in a time Δt is given by:

$$\Delta N = -\lambda N \Delta t$$

where λ is called the decay constant. We can integrate this expression to get the relationship

$$N(t) = N_0 e^{-\lambda t}$$

where N_0 represents the number of radioactive nuclei at time t = 0.

a) Show that the time taken for N(t) = $N_0/2$ (known as the half-life), is given by $T_{1/2} = (\ln 2)/\lambda$.

Potassium-argon (K-Ar) dating is used in earth science to determine the age of a rock sample. The potassium isotope ${}^{40}_{19}\mathrm{K}$ is radioactive and decays over time to the argon isotope ${}^{40}_{18}\mathrm{Ar}$. When a rock is molten the argon gas is released into the atmosphere but as the rock cools and crystallizes, the daughter argon atoms are trapped within the rock matrix. Time since crystallization is calculated by measuring the ratio of the amount of ${}^{40}_{18}\mathrm{Ar}$ accumulated to the amount of ${}^{40}_{19}\mathrm{K}$ remaining.

The two possible nuclear reaction for ${}^{40}_{19}\mathrm{K}$ to change to ${}^{40}_{18}\mathrm{Ar}$ are:

$${}^{40}_{19}\mathrm{K} + {}^{0}_{-1}\mathrm{e} \rightarrow {}^{40}_{18}\mathrm{Ar} + \text{ a neutrino}$$

$${}^{40}_{19}\mathrm{K} \rightarrow {}^{40}_{18}\mathrm{Ar} + {}^{0}_{+1}\mathrm{e} + \text{ a neutrino}$$

b) Describe the physical process that is occurring in these reactions.
c) Which of the two is the more likely to occur?
d) Most of the potassium ${}^{40}_{19}\mathrm{K}$ (89.1%) decays to ${}^{40}_{20}\mathrm{Ca}$. What is the equation for this reaction?

The *approximate* ratio of the amount of ${}^{40}_{18}\mathrm{Ar}$ to that of ${}^{40}_{19}\mathrm{K}$ is directly related to the time elapsed since the rock was cool enough to trap the argon by the following equation:

$$t = \frac{t_{1/2}}{\log_e(2)} \log_e\left(\frac{K_f + \frac{Ar_f}{0.109}}{K_f} \right)$$

Where $T_{1/2}$ is the half-life of ${}^{40}_{19}\mathrm{K}$, K_f is the amount of ${}^{40}_{19}\mathrm{K}$ remaining in the sample, and Ar_f is the amount of ${}^{40}_{18}\mathrm{Ar}$ found in the sample.

e) If a rock sample has remained undisturbed for 50 million years and the half-life of ${}^{40}_{19}\mathrm{K}$ is 1.248 x10^9 years, what ratio of Ar/K should be measured in the sample?