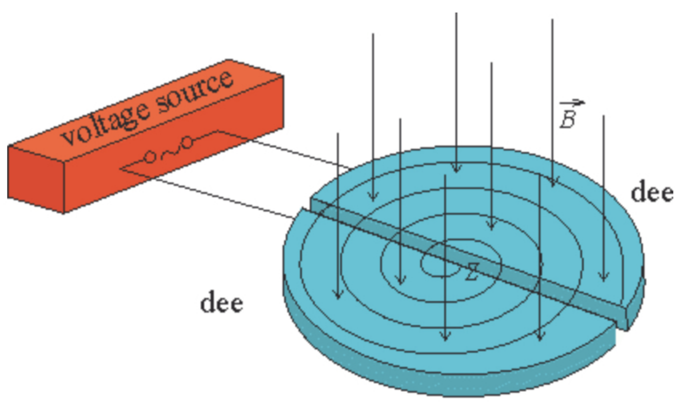
Problem Set No. 2

UBC Metro Vancouver Physics Circle 2018

March 15, 2018

Problem 1

A cyclotron is a type of particle accelerator invented by Ernest O. Lawrence in 1934. However, it is still in use today for research and medical purposes. Referring to the figure below, cyclotrons are made up of two semi-cylinder conductors that are opened on the straight side. These D-shaped conductors are made of a non-ferromagnetic material and are called "dees". The dees are placed in a homogenous magnetic field with a great magnetic induction that is perpendicular to the conductors. An alternating voltage is brought into a narrow gap between the conductors. If a charged particle enters the electric field between the dees near the center of the cyclotron, it is accelerated and gains a velocity that runs perpendicularly to the magnetic field inside one of the dees.



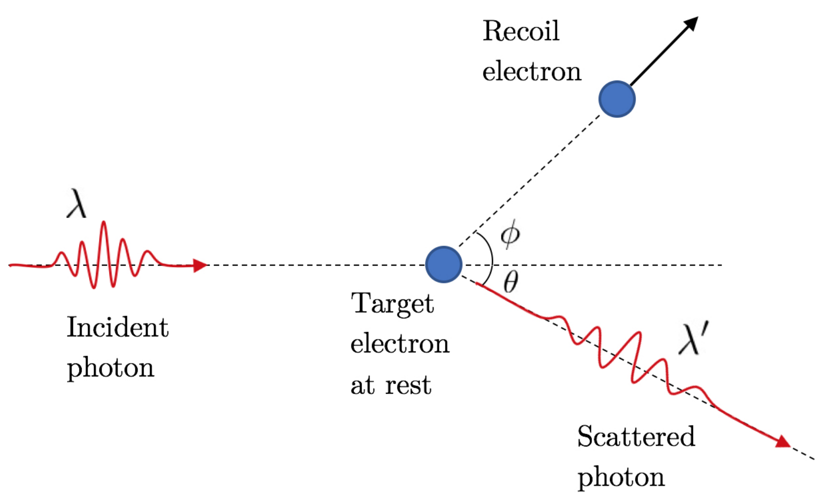
The magnetic field forces the particles to travel in a spiral path. The energy is applied to the particles as they cross the gap between dees in the cyclotron and so the particles are accelerated. This effect increases the radius of the circle and so the path is spiral. With no change of energy, the charged particles follow a circular path. A charged particle will gain its maximum speed when the radius of its path reaches the radius of the cyclotron.

(a) Show that the period of rotation of a particle with charge q is independent of its speed or path's radius, and only depends on the particle's mass and charge, and the magnetic field strength, $|\vec{B}|$.

(b) A cyclotron contains an alternating voltage source of 50 kV and a uniform magnetic field strength of 1.0 T. If a proton reaches the very edge of the cyclotron and exits with 25 MeV of energy, what is the diameter of each dee and how many times did the proton go through the dees before it exited the cyclotron?

Problem 2

Compton scattering, discovered by Arthur H. Compton, is the scattering of a photon by a charged particle, usually an electron. Compton explained and modeled the data by assuming a particle nature for light and applying conservation of energy and conservation of momentum to the collision between the photon and the electron. The scattered photon has a lower energy since part of the incident photon's energy is transferred to the recoiling electron. Since its energy is lower, its wavelength must increase due to the Planck relation, $E = hf = \frac{hc}{\lambda}$; this shift in wavelength is called the Compton Effect.



If λ is the wavelength of the incident photon, λ' is the wavelength of the scattered photon, we achieve the Compton relation:

$$\lambda' - \lambda = \Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

where h is Planck's constant, m_e is the electron's mass, and θ is the scattering angle.

Using conservation of momentum, derive the following relation:

$$\cot\frac{\theta}{2} = \left(1 + \frac{hf}{m_e c^2}\right)\tan\phi$$

Problem 3

A physicist wants to explore a strange body of water. The reason this "ocean" is strange is because it has a floor that is perfectly slanted at some angle; to the physicist's knowledge, there is no end to the depth of this ocean. It is also relatively quite boring, as there is nothing except water — no fish, no aquatic plants, not even rock formations of any kind. However, this strange and boring ocean is the perfect place to do experiments!

The physicist pilots a boat at constant speed, v_B. He has a device attached to the boat that sends and receives signals; signals are sent straight down at constant speed, v_S. These signals also have a strange effect due to their dispersion in all directions when making contact with a solid surface. As a result, when a signal is sent down making contact with the ocean floor, one reflecting signal can make it back perfectly to the boat where it is detected by the device. Once a signal is received, the device immediately sends another one straight down.

We may assume the following:

(1) the ocean floor is tilted down at angle θ below the horizontal,
(2) $v_S \gg v_B$, and
(3) no matter how weak the returning signal, it is always detected by the device.

(a) If the physicist sends the first signal after travelling some distance across the ocean, show that

$$\frac{d_{n+1}}{d_n} = K$$

where K is a constant, only in terms of v_S, v_B, and θ; d_n is the distance the boat travels between the time it sends and receives the n^{th} signal. Ultimately, this suggests that the rate at which travelled distances increase is always constant for consecutive signals only.

(b) Using your answer in Part (a), find the expression for the **total** distance, D_n, the boat travels (since it sends its first signal) for the n^{th} signal sent and received. Your expression should be in terms of n, v_S, v_B, θ, and d_0, where d_0 is the distance the boat has travelled (from the beach) before it sends its first signal.

For both parts above, you can refer to the diagram below.