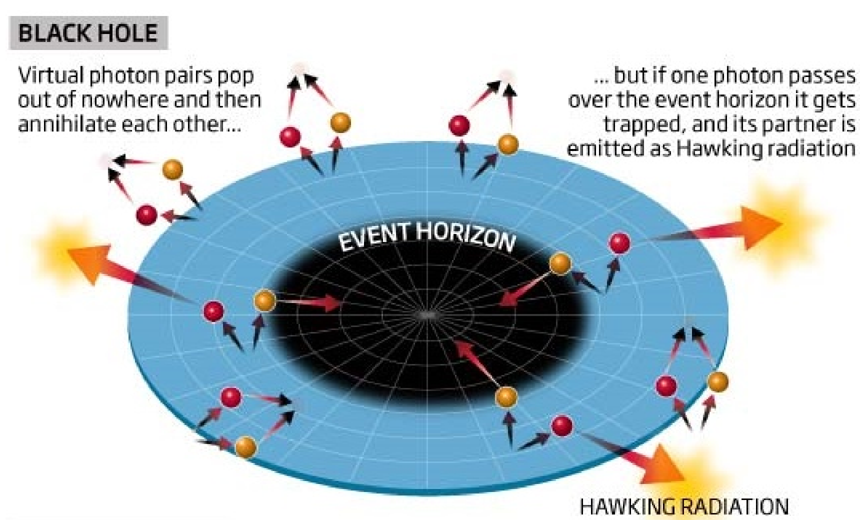
Problem Set No. 3

UBC Metro Vancouver Physics Circle 2018

April 5, 2018

Problem 1 — Hawking Radiation

The late Stephen Hawking made extraordinary contributions to the field of cosmology and astrophysics. Although he recently passed away, his genius and outstanding work will forever apply to our present understanding of the universe. One of his outstanding contributions was Hawking Radiation; one of the first attempts of uniting general relativity with quantum mechanics. Also known as Hawking-Bekenstein Radiation, this interesting hypothesis predicts the release of blackbody radiation from black holes due to quantum effects near the event horizon. Although it is yet to be detected, it is widely accepted. Due to quantum phenomena, particle pairs pop into existence randomly throughout space; these particles are opposite in nature — for example, if one were to have a positive mass, the other has negative mass. Although the pair comes into existence out of nowhere, it is very short-lived as the particles soon after collide back into each other and annihilate one another. Near a black hole, something interesting happens.



When a photon pair is created near the event horizon of a black hole, one will go off past the event horizon, and thus will get trapped due to the extreme gravitational pull. However, the other will diverge and go away from the event horizon, thus able to escape the black hole. This particle is emitted as radiation. Thus, under the assumption of pure photon emission (no other particles are emitted) and under the assumption that the horizon is the radiating surface, Stephen Hawking and Jacob Bekenstein predicted the luminosity of a black hole is inversely proportional to the square of its mass.

Using dimensional analysis, determine an equation for this predicted luminosity by only using Planck's constant (h), speed of light (c), Newton's gravitational constant (G), and mass of the black hole (M). Black holes can have thousand to billion times the mass of our Sun! Even if we consider a very "light" black hole of 1 solar mass ($M_\odot = 2 \times 10^{30}$ kg), why is it difficult to detect Hawking Radiation if the proportionality constant is found to be small?

Problem 2 — Muon Decay

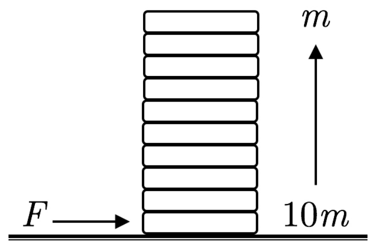
Muons are elementary particles similar to electrons, but with a much greater mass; as a result, they are classified within the lepton family. They are very unstable with a mean lifetime of 2.2 μs. As cosmic rays interact with the upper atmosphere, muons are generated in the process and fall towards Earth. Due to their short lifetime, one would not expect to observe muons at sea-level as they will decay much before that. However, we do detect muons! How is this possible? Since muons travel very close to the speed of light, relativistic effects occur, namely time dilation and length contraction. For the muon's reference frame, its length travelled becomes significantly compressed; in a stationary observer's reference frame, time for the muon significantly slows down, making its decay time significantly longer. Overall, due to these effects, it becomes possible for muons to reach Earth and be observed.

(a) Assuming the thickness of the atmosphere on Earth is ∼10 km and the average lifetime in a muon's rest frame is 2.2 μs, calculate the speed of muons as a multiple of the speed of light, c, if they just reach sea-level before they decay.

(b) Using your answer in part (a), determine the minimum energy needed for a muon to just reach sea-level if its rest mass is 105.7 MeV/c^2.

Problem 3 — Stacked Blocks

10 rectangular blocks with masses $m, 2m, 3m, ..., 10m$ are stacked on top of each other in decreasing order of mass. The entire ensemble lies on a frictionless surface. A force F is exerted on the bottom block.



If the coefficients of static friction between any two blocks are just enough to prevent slipping, which of the following is true?

A) $\mu_1^2 > \mu_2^3 > ... > \mu_9^{10}$

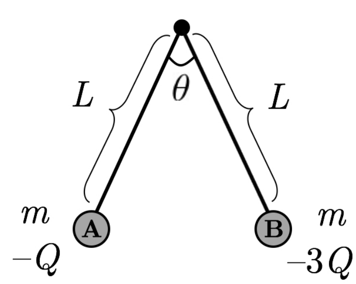
B) $\mu_1^2 = \mu_2^3 = ... = \mu_9^{10}$

C) $\mu_1^2 < \mu_2^3 < ... < \mu_9^{10}$

Where μ_i^j is the coefficient of static friction between blocks i and j.

Problem 4 — Two Charged Spheres

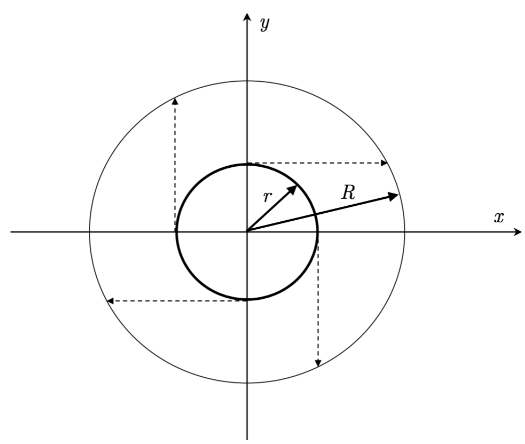
Two metal spheres, A and B, each with mass m are both attached to a fixed point by strings of length L. Sphere A is given a charge of $-Q$ and sphere B is given a charge of $-3Q$ so that they very slightly repel each other. Give an expression for the equilibrium angle θ between the two spheres. Since the electric force will be weak, you may use $\sin(\theta) \approx \theta$ and $\tan(\theta) \approx \theta$ as small angle approximations to find θ.



Problem 5 — Probability of Scoring a Goal

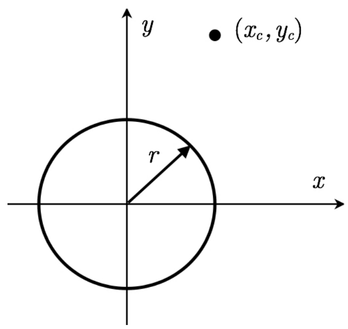
A ball, connected to a light string, is swinging in circular motion on a tabletop with a coefficient of friction, μ. As the ball follows a clockwise circular path of radius r, completing one revolution in T seconds, the light string snaps and the ball goes off at a tangent. Due to friction, however, the ball comes to rest eventually.

(a) If the string snaps at any position, the ball will always travel tangentially at a fixed distance before it comes to rest, due to the same frictional force. Since the circular path the ball follows before the string snaps is symmetric, the traced path of all the points at which the ball stops is also a circle. Let us call this latter circle the "bound circle" and let us label its radius R. If we orient the initial circle at the origin, we get:



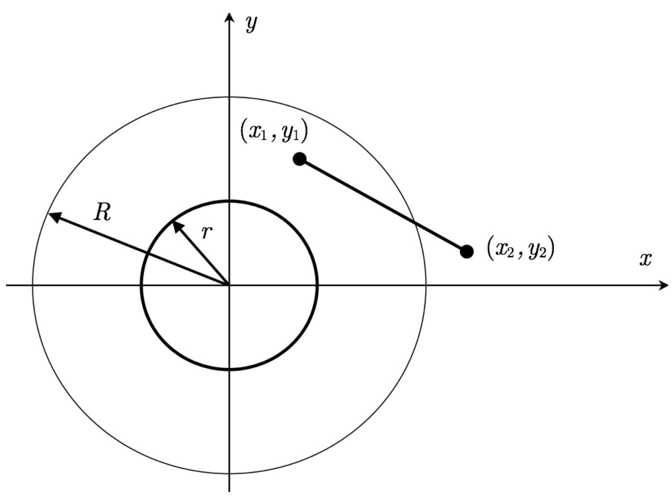
The dashed lines are tangents taken at test points to demonstrate that their fixed lengths give rise to the bound circle – again, note that the ball is travelling clockwise. Find the radius of the bound circle, R, in terms of only r, μ, g, and T, where g is acceleration due to gravity.

(b) Find an equation for the x-coordinates on the circle $x^2 + y^2 = r^2$ such that their tangents include an exterior point (x_c, y_c), which lies in the region $x^2 + y^2 > r^2$. Your equation should only be in terms of r, x_c, and y_c. Refer to the diagram below.



(c) Let's assume there is a "goal" that is enclosed linearly by posts at points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$. The goal posts are fixed according to the following rules:

(1) the point P_1 is always between the initial and bound circles,
(2) the point P_2 is always outside of the bound circle,
(3) $x_1 < x_2$,
(3) $y_1 > r$ while $y_2 < r$.
(4) and $\{x_1, x_2, y_1, y_2\} > 0$.



If the ball is travelling clockwise on the initial circle and the light string snaps completely randomly, what is the expression for the **maximum** probability that the ball will score a goal? Scoring a goal is defined as minimally making contact with the goal line; the ball will also score a goal if it makes contact with any goal post. The maximum probability expression should only be in terms of r, μ, g, T, x_1, x_2, y_1, and y_2.

Note: The answer has multiple variations. Answering one of these variations will suffice.