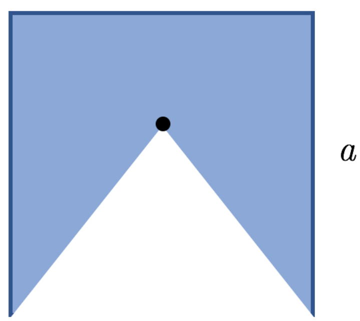
Problem Set No. 4

UBC Metro Vancouver Physics Circle 2018

April 19, 2018

Problem 1 — Balancing with Geometry

An isosceles triangle is to be cut from a square plate of uniform density ρ and width a, as shown below. What is the height of the removed isosceles triangle such that the resulting shape balances at the tip of the triangle, when holding the plate from the bottom (i.e. the plate is flat horizontally)? Your answer should only be in terms of a.



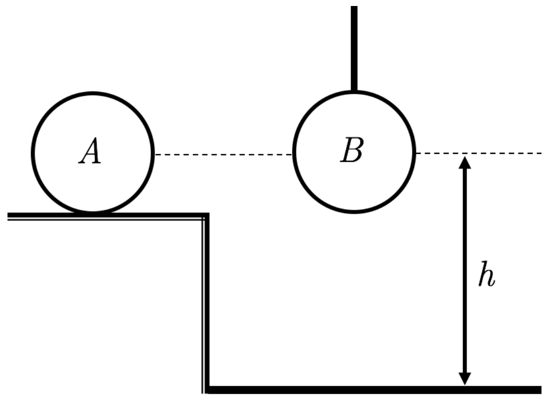
Problem 2 — Ocean Surface

The average ocean depth on Earth is approximately 3.6 kilometres. Given that 1 litre of water contains approximately 55 moles of H_2O, what fraction of water molecules in the ocean are located directly on the surface (i.e. there are no water molecules directly above them)?

Note that 1 mole = 6.0×10^{23} molecules.

Problem 3 — Heating Metal Spheres

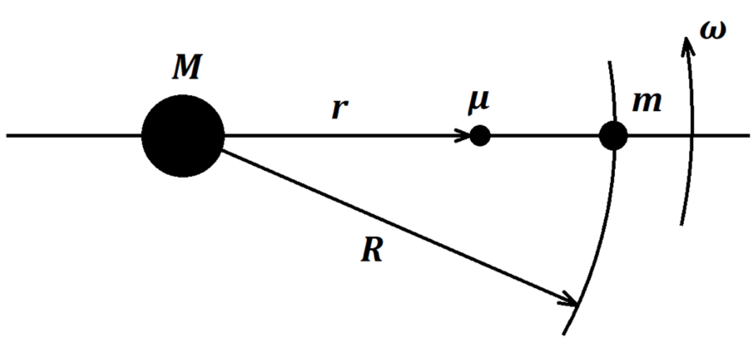
Consider two identical metal spheres A and B, both at a height h from the ground. A sits on a flat surface while B hangs from a string. A blowtorch is used to heat both metal spheres by the same amount ($Q_A = Q_B$). After heating, which sphere will have a higher temperature?



Note that an object's thermal energy is given by $E_T = mC\Delta T$, where m and C are the object's mass and heat capacity, respectively, and ΔT is the change in temperature. Assume both the surface and string are perfect insulators; therefore, you can neglect any form of heat loss.

Problem 4 — Lagrange Points

Consider the following planetary system consisting of a planet of mass m in a circular orbit of radius R around a star of mass M (assume M is sufficiently larger than m to neglect its movement).



(a) Find ω, the angular frequency of the planet, in terms of G, M, and R.

(b) We want to place a satellite of mass μ in a circular orbit around M such that it also orbits with an angular frequency of ω. The satellite is positioned along the line connecting m and M and has an orbital radius of r. Write an equation involving r that must be satisfied for the satellite to be at equilibrium (make sure to consider all three possible regions, left of M, between m and M, and right of m). Use substitutions $a = \frac{m}{M}$ and $x = \frac{r}{R}$ to simplify the equation and write it only in terms of a and x.

(c) The solutions to this equation are called the Lagrange Points. How many Lagrange points are there along the line connecting m and M? Justify your answer.

(d) Consider the solution of x to the right of m which would shield the satellite from the star's brightness and allow it to make better observations. What would this value of x be were m equal to zero? Now let a be a very small number and find x in terms of a using numerical approximations.

Hint: For a small a, we can approximate: $(1 + a)^n \approx 1 + na$.

(e) For the Earth and Sun system, the value of $a = 3 \times 10^{-6}$. How close would the satellite be to the Earth? How many times farther than the moon is this? (AU = 150 million km, Moon orbital radius = 385000 km)

(f) An important factor to be considered in putting a satellite in such orbit is the stability of the orbit. The stability can be studied by making small changes to the position and studying the behavior of the system. Is the solution obtained above stable? Justify your answer.