Problem Set No. 5

UBC Metro Vancouver Physics Circle 2018

May 17, 2018

Problem 1 — The Equilibrium Enigma

A physics student is to distinguish 3 cases apart from each other, set up by her professor. In each case, a solid ball of mass m, with the same dimensions and density, is hanging by a spring while submerged in some solution (see Figure 1).

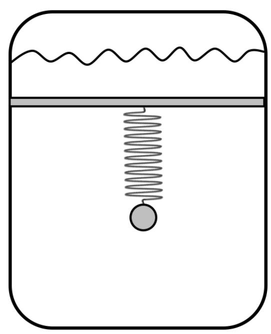


Figure 1: The apparatus.

However, what set these 3 cases apart are (1) the types of solution these objects are submerged in and (2) the strength of each spring. Assume that in all cases each ball is heavier than the solution and is perfectly in balance as it hangs from the spring. In reference to Figure 2, which of the following options correctly distinguishes the cases from one another if

(1) Case 2 and Case 3 share identical slopes,
(2) Case 1 and Case 3 share identical x-intercepts,
(3) and Case 1 and Case 2 share identical extrapolated y-intercepts?

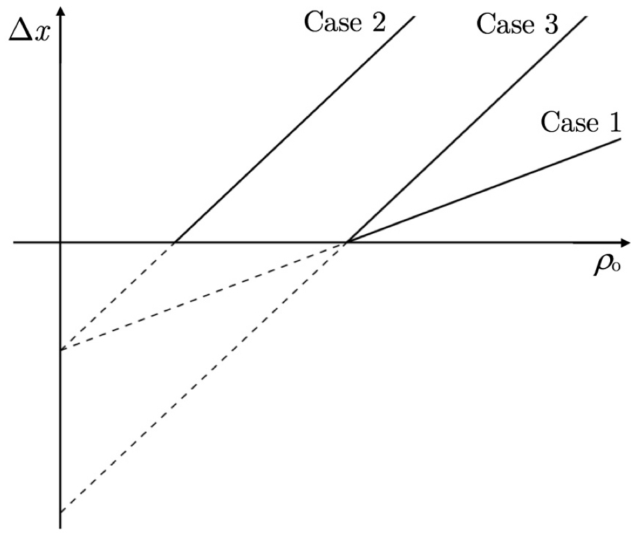


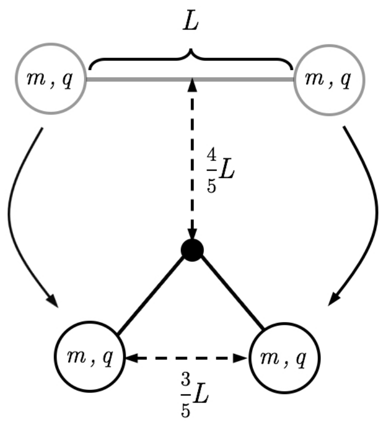
Figure 2: The extrapolated theoretical linear plots of the three cases, relating the length of spring expansion to the **density of the object**.

	Case 1		Case 2		Case 3	
	Solution	Relative Strength of k	Solution	Relative Strength of k	Solution	Relative Strength of k
A.	Water	Medium	Ethanol	Poor	Water	Poor
B.	Ethanol	Medium	Water	Poor	Ethanol	Poor
C.	Water	Poor	Ethanol	Medium	Water	Medium
D.	Ethanol	Poor	Water	Medium	Ethanol	Medium
E.	Ethanol	Medium	Ethanol	Poor	Water	Poor

NOTE: As mentioned in the question, the mass, size, and density of the objects are all the same. However, as the density of the objects increase in each case, the spring expansion is unique due to the different spring and solution properties.

Problem 2 — Spherical Charges: Reloaded

Two identical spheres each with mass m and charge q, are attached by a string of length L. They are then dropped from a height of $h = \frac{4}{5}L$ above a rod centred at the middle of the string. When the string hits the rod, the spheres approach each other until they reach a minimum distance of $\frac{3}{5}L$. Using conservation of energy, determine the numerical value ratio $\frac{F_e}{F_g} = \frac{k\frac{q^2}{L^2}}{mg}$.



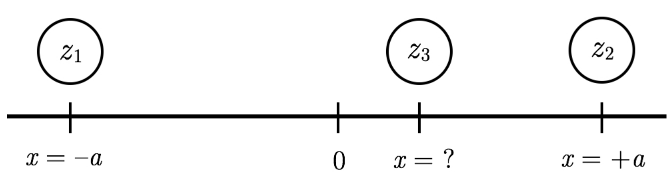
Problem 3 — The Balancing Act

In physics, many forces between two particles can be written in the form

$$F = Az_1z_2r^b$$

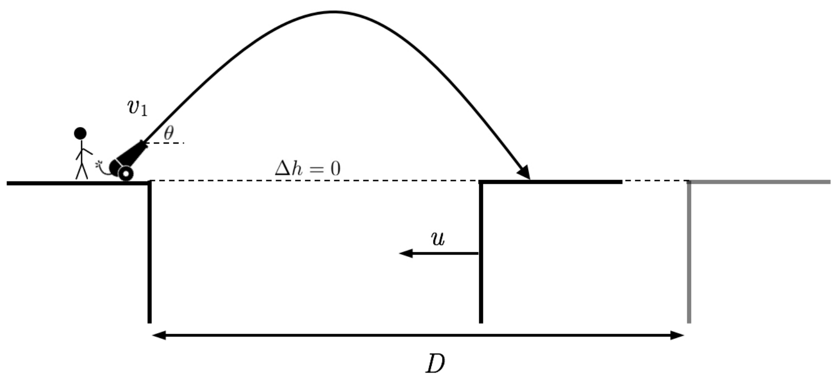
where z_1 and z_2 are an intrinsic property of the particles. For example, if z_1 and z_2 are masses, $A = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}, and $b = -2$, then we have the force of gravity.

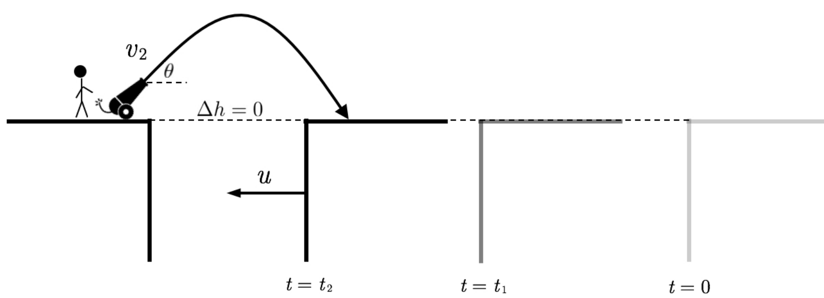
Suppose a new force of this form ($F = Az_1z_2r^b$) is discovered. If we place a particle with z_1 at $x = -a$ and a particle with z_2 at $x = +a$, the system will not be in equilibrium as the two particles will either attract or repel each other. However, we may be able to add a third particle z_3 at some position x to maintain equilibrium. Determine z_3 and x.



Problem 4 — The Cannon Conundrum

A physicist is standing on a stationary platform with a cannon set at a fixed angle, θ, above the horizontal ($0 < \theta < \pi/2$). There is another platform, separate from the first, that is moving towards the physicist at a constant speed, u; this second platform has the same height as the platform the physicist is standing on. The physicist wants to shoot a cannonball such that it lands on the moving platform. After the first shot lands, he immediately shoots another cannonball such that it lands on the same spot on the moving platform as the first shot. After the second shot lands, he immediately shoots the third such that it lands on the same spot on the moving platform again; this process continues as described.





Let's assume that the original distance between the two platforms at time $t = 0$, when the first shot is just about to be made, is D. Let's also assume that the angle for the cannon, θ, never changes. As a result, the only factor that is changing, for each shot that is taking place, is the speed of the cannonball exiting the muzzle.

If v_n represents the muzzle speed of the cannonball for the n^{th} shot, find $v_{n+1}(v_n)$; in other words, find the muzzle speed of the $(n+1)^{\text{th}}$ shot as a function of the muzzle speed of the n^{th} shot. Such a formula is recognized in mathematics as a recursive function, where a value is determine by a previous one.