Solutions to Problem Set No. 2

UBC Metro Vancouver Physics Circle 2018-2019

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Problem 1 — Discovering Antimatter

1. The Lorentz force law tells us that the particle is subject to a constant force of magnitude $F = Bqv > 0$. The force will be normal to the direction of motion, acting centripetally and causing the particle to move in a circle. To find the radius, we use $a = v^2/R$:
$$a = \frac{F}{m} = \frac{Bqv}{m} = \frac{v^2}{R} \quad \Longrightarrow \quad R = \frac{mv}{Bq}.$$
Finally, by the right-hand rule, a positively charged particle will experience a force to its left, causing it to move around the circle anticlockwise (seen from above); similarly, a negatively charged particle will move clockwise.

2. From the previous question, the particle's radius of curvature will get smaller as it slows down. This tells us the particle in the image is moving from bottom to top. (Being able to tell which the particle is going is why Anderson added the plate!) Since its path curves in the anticlockwise sense, it must be positively charged.

3. The radius of the track is comparable to the radius of the chamber, $r \approx 0.1\,\text{m}$. This leads to momentum
$$p = mv = BqR = 1.7 \times 0.1 \times (1.6 \times 10^{-19})\,\frac{\text{kg} \cdot \text{m}}{\text{s}} \sim 10^{-20}\,\frac{\text{kg} \cdot \text{m}}{\text{s}}.$$
This is considerably smaller than the momentum a proton would need to create the trail seen in the photograph. This only leaves one option: it is the positron, the positively charged evil twin of the electron!

Problem 2 — Rockets and Space Elevator

1. The gravitational acceleration is given by Newton's law of gravitation:

$$a = \frac{GM}{r_{\text{geo}}^2}.$$

The centrifugal acceleration is

$$a = \frac{v^2}{r_{\text{geo}}} = \omega^2 r.$$

Equating the two, we find

$$r_{\text{geo}}^3 = \frac{GM}{\omega^2}.$$

Since $\omega \propto 1/T$, where T is the period of the orbit, Kepler's third law is obeyed.

2. Since the whole elevator is geostationary, it rotates with angular frequency ω. At radius r_{esc}, the speed is $v = \omega r_{\text{esc}}$. We recall that the gravitational potential is $U = -GMm/r$. Finally, we can determine r_{esc} by demanding that the total energy vanish:

$$E = U + K = m\left(\frac{1}{2}\omega^2 r_{\text{esc}}^2 - \frac{GM}{r_{\text{esc}}}\right) = 0 \implies r_{\text{esc}}^3 = \frac{2GM}{\omega^2} = 2r_{\text{geo}}^3.$$

3. Treat the rod as concentrated at its centre of mass at radius R. In order for the rod and the satellite to have the same angular velocity, we require the forces in the rotating reference frame to balance:

$$\omega^2[(R-L)+(R+L)] = 2R\omega^2 = GM\left[\frac{1}{(R-L)^2} + \frac{1}{(R+L)^2}\right] = \frac{2GM(R^2+L^2)}{(R^2-L^2)^2}.$$

Rearranging, we find that

$$\frac{GM}{\omega^2} = \frac{R(R^2-L^2)^2}{(R^2+L^2)}.$$

If $L \ll R$, then $(L/R)^2 \ll 1$ and hence

$$\frac{1}{R^2+L^2} = \frac{1}{R^2(1+L^2/R^2)} \approx \frac{1}{R^2}\left(1 - \frac{L^2}{R^2}\right),$$

using our approximation $1/(1+x) \approx 1-x$. It follows that

$$\frac{GM}{\omega^2} \approx \frac{1}{R}(R^2-2L^2)(R^2-L^2) \approx R^3 - 3RL^2.$$

Comparing to the radius of the geostationary orbit, we find

$$r_{\text{geo}}^3 \approx R^3 - 3RL^2,$$

which implies that $r_{\text{geo}} < R$.

Problem 3 — The Quantum Hall Effect

1. The current I is the rate at which charge flows through a cross-section of the conductor. Concretely, if ΔQ moves through the cross-section in time Δt, then $I = \Delta Q/\Delta t$. Take a cross-section slice of the conductor at some fixed time. The charges in the slice move at speed v_x, so in time Δt, they drag out a volume $\Delta V = Av_x\Delta t$. All the charge in this volume will move through a fixed cross-section over time Δt, so we just need to find the charge in the volume ΔV. This is simply the volume multiplied by the charge and density of the carriers:

   $$\Delta Q = nq\Delta V = Anqv_x\Delta t.$$

   Thus, we have the current:

   $$I = \frac{\Delta Q}{\Delta t} = Anqv_x.$$

2. From the Lorentz force law, the force on charges in the y direction is

   $$F_y = qE_y - qv_xB.$$

   The transverse field is defined as the field needed to balance the magnetic force, so that

   $$F_y = qE_y - qv_xB = 0 \quad \Longrightarrow \quad E_y = v_xB.$$

   Using $V_{\text{H}} = wE_y$ and the results of part (1), we find that

   $$V_{\text{H}} = wE_y = wv_xB = \frac{IwB}{Anq} = \frac{IB}{bnq},$$

   using $A = bw$.

3. The sign of the Hall voltage depends on the sign of the charge q. All Hall needed to do was measure the voltage across the conductor to find the sign of the charge carriers!

Choosing $B > 0$, Hall observed that $V_{\mathrm{H}} < 0$. It follows that $q < 0$, i.e. the charge carriers are negative.

4. Denote units of mass, length and time by M, L, T respectively. Using $K = mv^2/2$ to get the units of energy, we have

$$[h] = [\text{energy}][\text{time}] = \frac{ML^2}{T^2} \cdot T = \frac{ML^2}{T}.$$

On the other hand, from the Lorentz force law and $F = ma$, the units of magnetic field are

$$[Bq] = \frac{[\text{force}]}{[\text{velocity}]} = \frac{ML/T^2}{L/T} = \frac{M}{T}.$$

Thus, dividing h by Bq gives something with units length squared. Taking the square root, we obtain the magnetic length:

$$\ell_B = \sqrt{\frac{h}{Bq}}.$$