Problem Set No. 2

UBC Metro Vancouver Physics Circle 2018-2019

November 29, 2018

Problem 1 — Discovering Antimatter

In 1928, Paul Dirac made a startling prediction: the electron has an evil twin, the *anti-electron* or *positron*. The positron is the same as the electron in every way except that it has positive charge $q = +e$, rather than negative charge $q = -e$. In fact, *every* fundamental particle has an evil, charge-flipped twin; the evil twins are collectively called *antimatter*.[1]

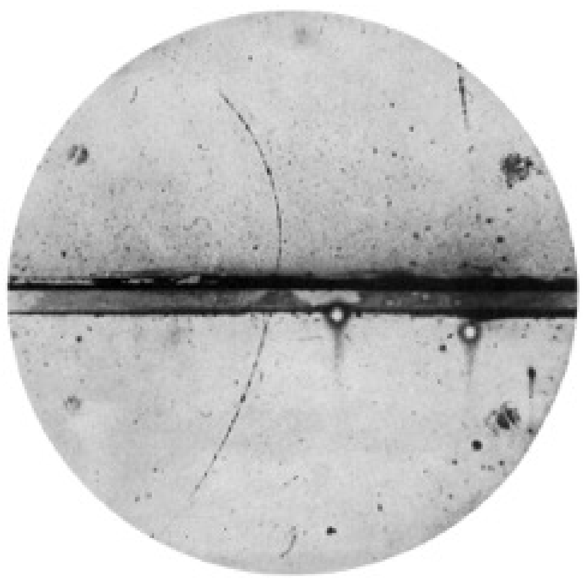

Figure 1: The mysterious trail in Carl Anderson's cloud chamber.

[1] You may think it is a unfair to call antimatter "evil", but if you met your antimatter twin, hugging them would be extremely deadly! You would annihilate each other, releasing the same amount of energy as a large nuclear bomb.

Experimentalist Carl Anderson was able to verify Dirac's prediction using a *cloud chamber*,[2] a vessel filled with alcohol vapour which is visibly ionised when charged particles (usually arriving from space) pass through it. In August 1932, Anderson observed the mysterious track shown above. Your job is to work out what left it!

1. A magnetic field $B = 1.7\,\mathrm{T}$ points into the page in the image above. Suppose that a particle of charge q and mass m moves in the plane of the picture with velocity v. Show that it will move in a circle of radius $R = mv/Bq$, and relate the sign of the charge to the motion.

2. The thick line in the middle of the photograph is a lead plate, and particles colliding with it will slow down. Using this fact, along with part (1), explain why the track in the image above must be due to a positively charged particle.

3. The width of the ionisation trail depends on what type of particle is travelling through the chamber and how fast it goes. The amount of ionisation in the picture above is consistent with an electron, but also an energetic proton, with momentum

$$p_{\mathrm{p}} \sim 10^{-16}\,\frac{\mathrm{kg}\cdot\mathrm{m}}{\mathrm{s}}.$$

   Can you rule the proton out?

[2] Cloud chambers are the modest ancestor of particle physics juggernauts like the Large Hadron Collider (LHC). Unlike the LHC, you can build a cloud chamber in your backyard!

Problem 2 — Rockets and Space Elevator

A space elevator is a giant cable suspended between the earth and a counterweight at the other end, orbiting the earth. Both the cable and counterweight are fixed in the rotating reference frame of the earth, and can be used to efficienctly transport objects from the surface into orbit, and also as a launchpad for rockets or satellites. Space elevators would completely revolutionise our access to space, and make large-scale projects like interplanetary travel to Mars a possibility.

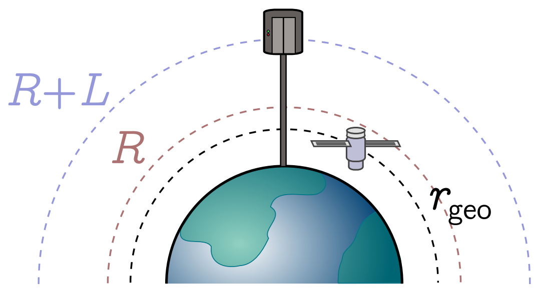

Figure 2: A satellite in geostationary orbit at radius r_{geo}. A space elevator connects a counterweight in low orbit to the surface via a cable of length $2L$. The cable's centre of mass lies at radius R, above r_{geo}.

1. To begin with, forget the cable, and consider a *geostationary* satellite orbiting at a fixed location over the equator.

   - Determine the radius r_{geo} of a geostationary orbit in terms of the mass of the earth M and angular frequency ω about its axis.
   - Confirm that r_{geo} obeys Kepler's third law, i.e. the square of the orbital period is proportional to the cube of the radius.

2. To make the space elevator, we now attach a cable to the satellite. The satellite acts as a counterweight, pulling the cable taut, but needs to move into a higher orbit in order to balance the cable tension. Provided this orbit is high enough, the space elevator will double as a rocket launchpad. Show that objects released from the elevator at $r_{\text{esc}} = 2^{1/3} r_{\text{geo}}$ will be launched into deep space.

   Hint: To launch a rocket, it needs to be travelling at escape velocity. This is the speed needed to leave the earth's gravity well with no gas left in the tank, i.e. the total energy (kinetic plus potential) vanishes.

3. The dynamics of the elevator itself are complicated, so we will consider a simplified model where the cable is treated as a rigid rod of length $2L$, with all of its mass concentrated at the centre, radius R. The counterweight is therefore at radius $R + L$.

   - Find the exact relationship between L, R, and the earth's mass M and rotational period ω.
   - Assuming $L \ll R$, show that the rod's centre of mass is further out than the geostationary radius r_{geo}. This somewhat counterintuitive result also holds for real space elevator designs! You may use the fact that, for $x \ll 1$,

$$\frac{1}{1+x} \approx 1 - x.$$

Problem 3 — The Quantum Hall Effect

Suppose we have a conductor made of a long, flat plate, with an electric field E_x running along its length. A magnetic field B points out of the conductor.

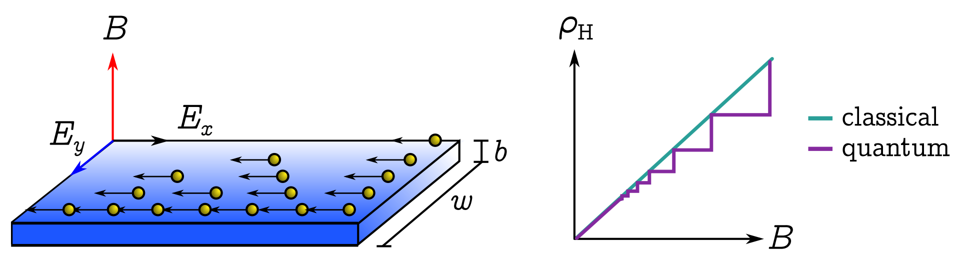

Figure 3: *Left.* A conductor with longitudinal electric field E_x and negative charge carriers. A magnetic field B (red) points out of the conductor pushing the carriers towards us. A transverse field builds up. *Right.* Classical vs quantum Hall resistivity as we vary the magnetic field.

Charges moving through the conductor are pushed to one side by the magnetic field, until the charge imbalance generates a *transverse field* E_y large enough to counteract the magnetic force. This phenomenon is called the *Hall effect*, and the corresponding voltage across the conductor the *Hall voltage* V_{H}.

1. Show that the current I in the x direction is related to the cross-sectional area A of the conductor

$$I = Anqv_x,$$

where v_x is the velocity in the x direction, q is the charge of the carriers, and n is the number of carriers per unit volume.

2. The transverse field and Hall voltage are related by $E_y = V_{\mathrm{H}}/w$, where w is the width of the conductor. Argue that
$$V_{\mathrm{H}} = \frac{IB}{nbq},$$
where b is the height of the conductor.

3. Edwin Hall discovered the effect in 1879, before anyone knew about the electron. How could he tell that charge carriers in metal were negative?

The Hall voltage depends the size and shape of the conductor. The *Hall resistivity* ρ_{H} is a new quantity we define which depends only on B and microscopic properties of the material:

$$\rho_{\mathrm{H}} = \frac{V_{\mathrm{H}}}{Ib} = \frac{B}{nq}.$$

It seems that by tuning the magnetic field, we could make the Hall resistivity anything we like. But if we switch on quantum mechanics, only certain values of ρ_{H} are allowed. This is called the *quantum Hall effect*. Note that the strength of quantum mechanical effects is governed by *Planck's constant*, $h = 6.63 \times 10^{-34}\,\mathrm{J}\cdot\mathrm{s}$.

4. Using dimensional analysis, combine h, B and q to find the *magnetic length* ℓ_B of the system. This characterises the effective strength of the magnetic field, which does not involve n.