Problem Set No. 7

UBC Metro Vancouver Physics Circle 2018-2019

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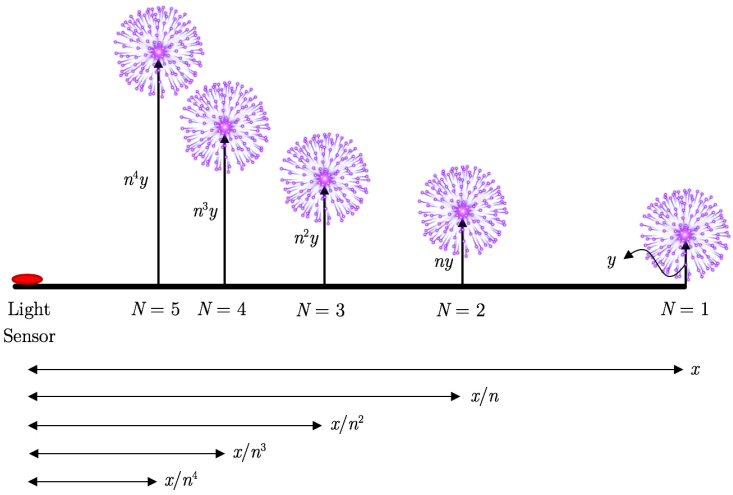
Problem 1 — Intense Fireworks

A light sensor is placed flat on the ground. At a horizontal distance x from the centre of the light sensor, a firework explodes when it travels a vertical height y. The light sensor then measures an intensity reading of I_1 for this $N = 1$ shot. When all the light from the first firework dissipates, the second firework ($N = 2$ shot) explodes at a horizontal distance $\frac{x}{n}$ from the centre of the light sensor immediately after it travels a vertical height ny. Then, a second intensity reading is made by the light sensor, measuring I_2. This cycle continues for the $N = 3, 4, 5, ..., N^{\text{th}}$ shot, each time with the horizontal distance decreasing by n and the vertical distance increasing by n.

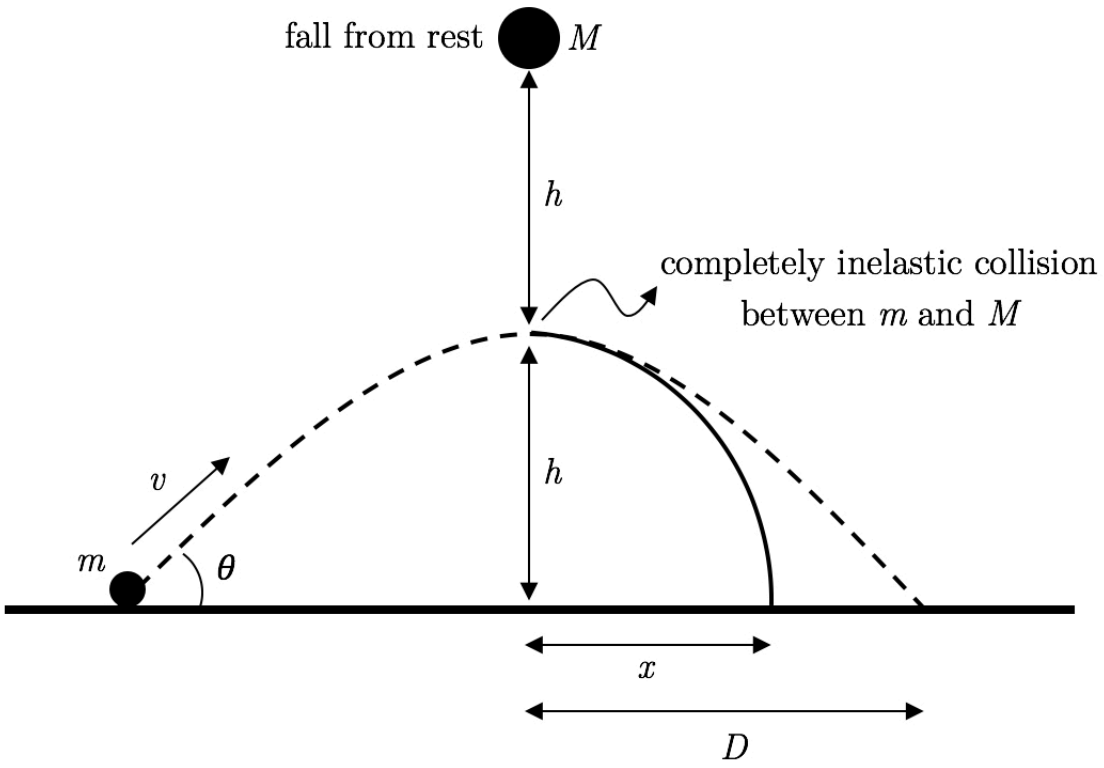


Show that consecutive intensity measurements decrease by a constant factor, n^2, when N is very large. That is, mathematically prove that the following is true:

$$\lim_{N\to\infty} \frac{I_{N+1}}{I_N} = \frac{1}{n^2}$$

Problem 2 — A Sticky Situation

A mass m undergoes projectile motion, where it is shot with velocity v at an angle of θ from the horizontal. It achieves a maximum height h and traverses a horizontal distance D from there to the ground. Now, let's assume a mass M is let go from rest at a height h right above the maximum point of m's trajectory. Mass M is let go such that when m reaches its maximum height, the two masses collide completely inelastically at that point. Then, the combined masses fall to the ground traversing a horizontal distance x from the point of contact to the ground.



1. If $\alpha = \dfrac{m}{M}$, find the expression for $\dfrac{x}{D}$ in terms of α only. You may ignore air friction.
2. If $f(\alpha) = \dfrac{x}{D}$, show that $f(\alpha)$ approaches 1 as α becomes very large.[1] In other words,

$$\lim_{\alpha \to \infty} f(\alpha) = 1$$

3. How many times more massive must m be compared to M such that $f(\alpha) = 0.5$?

[1] In other words, colliding mass M becomes increasingly insignificant to m's trajectory as m increases.