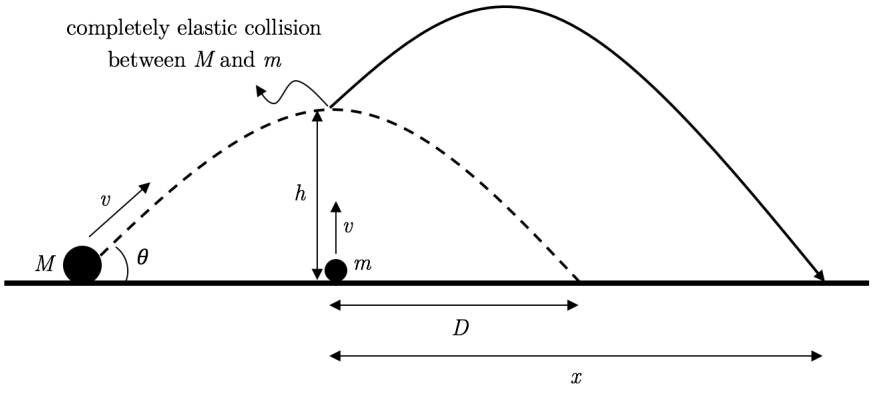
Problem Set No. 8

UBC Metro Vancouver Physics Circle 2018-2019

April 11, 2019

A *Non*-Sticky Situation

A mass M undergoes projectile motion, where it is shot with speed v at an angle of θ from the horizontal. It achieves a maximum height h and traverses a horizontal distance D from there to the ground. Now, let's assume a mass m is shot vertically upwards with speed v right below the maximum point of M's trajectory. Mass m is shot in such a way that when M reaches its maximum height, the two masses collide completely elastically at that point. Then, mass M undergoes a second projectile traversing a horizontal distance x from the point of contact to the ground. You may ignore air friction for all the questions below.



1. If v_1 and v_2 are the the vertical velocities of m and M after the collision, respectively, find the expressions for v_1 and v_2 both in terms of the variables m, M, v, and h. Then, find v_1 and v_2 both in terms of the variables m, M, v, and θ.
2. If $\alpha = \dfrac{m}{M}$, find the expression for $\dfrac{x}{D}$ in terms of α and θ only. In other words, find

$$f(\alpha, \theta) = \frac{x}{D}$$

3. If we call $f(\alpha, \theta)$ the ratio function, solve for α when $\theta = \dfrac{\pi}{4}$ and the ratio function is equal to 2.

4. Find $\alpha(\theta)$ for any angle when the ratio function is 2. For the ratio function to be equal to 2, find the maximum exclusive angle (the bound angle) in degrees that restricts θ to $0° < \theta < \phi$, where ϕ is <u>not</u> necessarily 90°. Ultimately, this means that when $\theta \geq \phi$, we can never get the ratio function to equal to 2 for any α value.

5. Find $\alpha(\theta)$ for any angle when the ratio function is equal to a value c, where $c > 1$. For the ratio function to be equal to c, find the maximum exclusive angle (the bound angle) in degrees, as a function of c, such that it restricts θ to $0° < \theta < \Phi(c)$. Then, use a graphing calculator or an online tool to graph $\Phi(c)$ for $1 < c < 50$. Ultimately, this function describes the exclusive upper bound of angle θ that allows the ratio function to reach any c for some α value.

6. If there is another mass m placed under M's trajectory in the second projectile, also being shot vertically upwards at speed v, prove that $0 < \alpha < 1$ is the only range of α values that guarantees a second collision between M and m. Refer to the diagram below.