

## UBC Physics Circle



Session 3: Solutions

November 14, 2019

### No Rush

1. The catch is that light follows a path that only *locally* takes the least time. Photons are unable to “look ahead” and plan out the quickest possible route. Instead, they follow paths that (from the information in their vicinity) seem to be as quick as possible. If you carefully examine the path through the prism, you will see that there is no tiny change you can make to it that will make it take a little bit less time. Changing it into the direct path will certainly reduce the time, but that is not a tiny change. This means photons do not need to take the globally shortest path; a path which is locally the shortest will do just fine.
2. Paths hitting the mirror are in a different “category” than paths that don’t hit the mirror. (Similarly, paths hitting 5 mirrors are in a different category from paths hitting 6 mirrors, and so on.) The two paths shown above are in different categories, and so they don’t “compete” with each other.

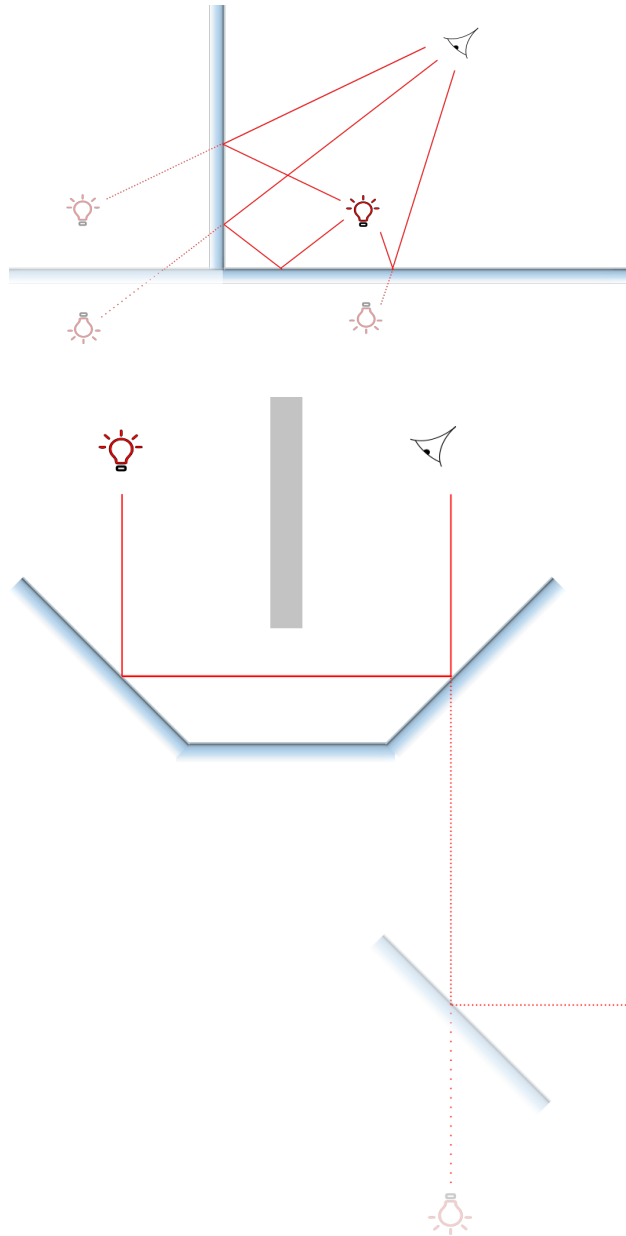
Why does light hitting the mirror put it into a separate category? Remember that in the answer to part 1 we said that a path is allowed as long as no tiny change can make it shorter. Well in this situation, converting the path from one that hits the mirror to one that does not hit the mirror is, in an important sense, not a tiny change. For a deeper explanation of the principle of least time and where it comes from, see the Feynman Lectures on Physics, Volume 1, Chapter 26.<sup>1</sup>

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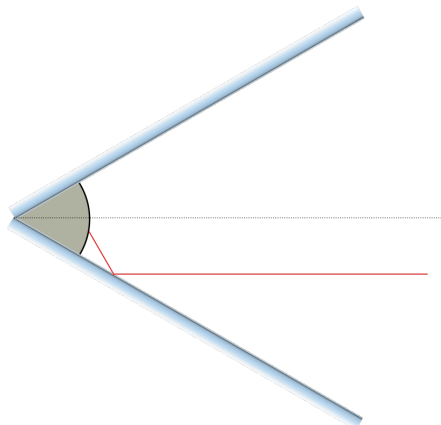
<sup>1</sup>[http://www.feynmanlectures.caltech.edu/I\\_26.html](http://www.feynmanlectures.caltech.edu/I_26.html)

## Method of Images

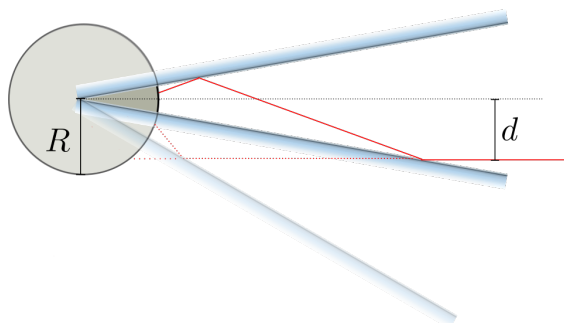
1. There are three images for the left setup and one for the right setup.



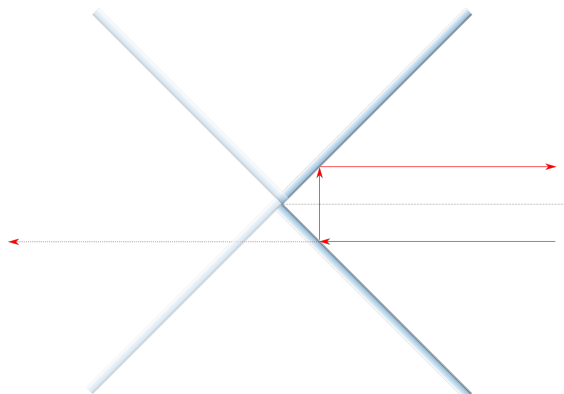
2. (a) Yes, the photon hits the detector after only one reflection.



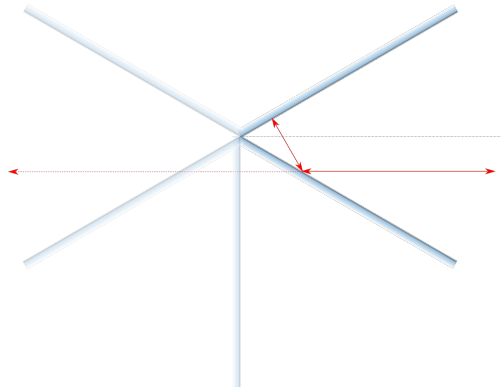
A useful trick is to follow the image of photon. It follows a path parallel to the horizontal. If the image (or image of the image, and so forth) hits the detector then the real photon will hit the detector. If  $d < R$  then the photon will hit detector. For  $R = 0.5\text{ m}$ ,  $d = 0.4\text{ m}$  the photon hits the detector no matter what  $\theta$  is.



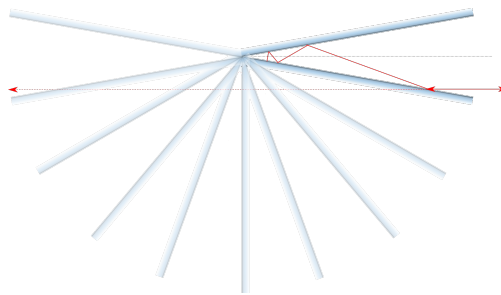
- (b) We can use the same trick to count the number of reflections ( $N$ ). If  $\theta = \frac{\pi}{4}$  then  $N = 2$ .



If  $\theta = \frac{\pi}{6}$  then  $N = 3$ . The photon leaves the mirror cone along the same path it entered.



If  $\theta = \frac{\pi}{18}$  then  $N = 9$ .



In general:  $\theta = \text{ceil} \left( \frac{\pi}{2\theta} - \frac{1}{2} \right)$  where the ceiling function "ceil" means to round up to the nearest integer.

## Mirage

1. The simplest model for refractive index is  $n(\rho) = m\rho + b$ . At zero density, the refractive index is 1, so  $b = 1$ . When  $\rho = \rho_0$ ,  $n(\rho_0) = 1.0003$ . This implies that  $m = 0.0003/\rho_0$ , yielding the equation  $n(\rho) = 1 + 0.0003(\rho/\rho_0)$ .
2. Rearranging the ideal gas law gives  $\rho = P/k_B T$  and hence  $\rho_0 = P_0/K_B T_0$ . If we assume  $P_0 = P$  (a good approximation for nearby bodies of air), then:  $\rho/\rho_0 = T_0/T$ . Then the equation for  $n(T)$  becomes

$$n(T) = 1 + 0.0003 \cdot \frac{298 \text{ K}}{T}.$$

3. The criterion for total internal reflection,  $\sin(\theta) = n(T)/n(298 \text{ K})$  implies

$$T = (298 \text{ K}) \left[ \frac{0.0003}{\sin(\theta)1.0003 - 1} \right].$$

We can calculate the angle using  $\theta = \arctan(200/1.5) = 89.57^\circ$ . Plugging this into our formula for  $T$  gives  $T = 329 \text{ K}$ , or about  $51^\circ \text{ C}$ . Incidentally, this is why you don't walk your dog on asphalt after the Sun has been beating down on it all day – it can often be hot enough to burn them.