## UBC Physics Circle

Session 3: Problems

November 7, 2019

The first problem provides a deeper understanding of the **principle of least time**. The second introduces the useful and intriguing **method of images**. And the last one is an application of what we learnt to explain a **mirage**.

No Rush
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Subtleties with the principle of least time

1. The following diagram shows two paths that light can take between A and B. One path is a straight line and the other involves refracting through a prism.
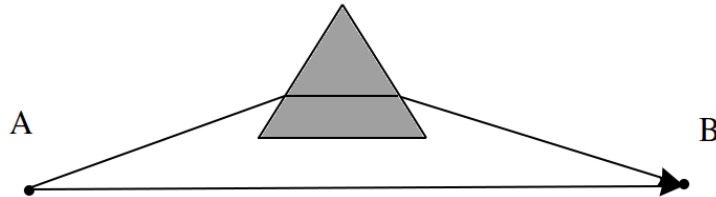
Both paths are physically possible: if you stand at point A and aim a laser directly at B, the light takes the lower path, but if you shine the laser through the prism at just the right angle, it will take the upper path. But the straight path obviously takes less time! This seems to contradict the principle of least time. Can you think of a change to our stated definition of principle of least time that resolves this contradiction?

2. Now consider two paths from A to B in the vicinity of a mirror: the direct route, and a second path which bounces off the mirror. Both paths are physically possible, but the straight path clearly takes less time. Is this consistent with your update to the principle of least time from the previous question? If not, can you tweak to the definition once more to resolve this contradiction?
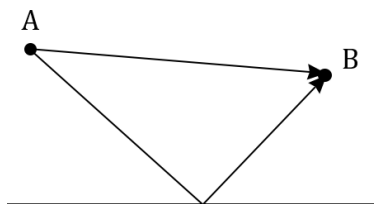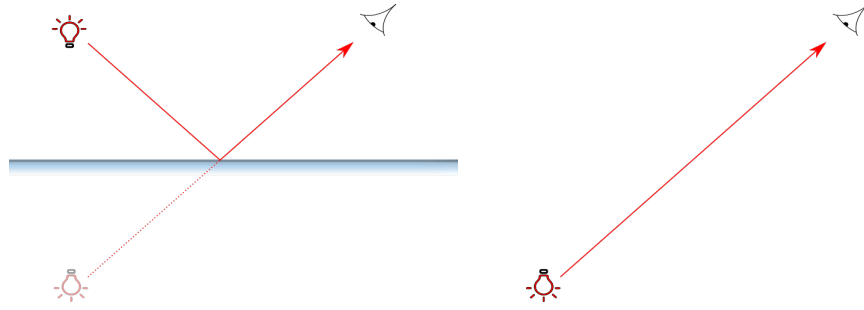

—Phillip Bement

Method of Images

A beautiful and handy technique

When a ray of light reflects off of a mirror and enters the eye, the observer sees the light source placed somewhere behind the mirror.

What the observer sees in the situation on the left is equivalent to the situation on the right. The "image" of the light source is behind the mirror.

1. How many images of the light bulb are there in each case below? Where are they located?
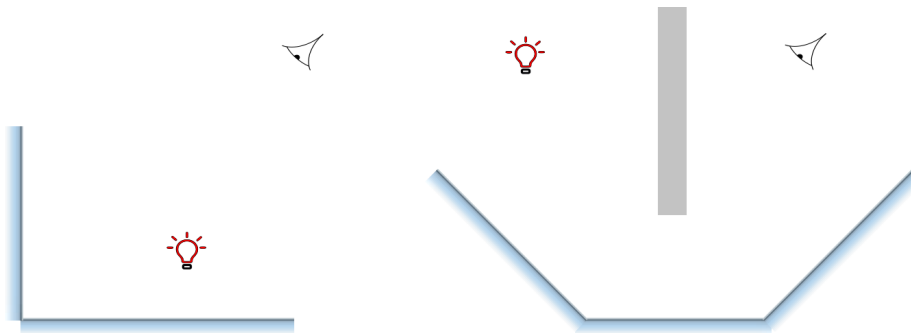
2. Two flat mirrors are connected at an angle of 2θ to form a mirror wedge. At the inner corner of the wedge, a photosensitive detector is placed, which is a section of a cylinder of radius R. An incoming photon may reflect off of either mirror many times, possibly hitting the detector or reflecting back out never to be seen again.

    (a) Consider an incoming photon, travelling parallel to the horizontal and displaced by distance d from it. Will the photon be detected by the detector?
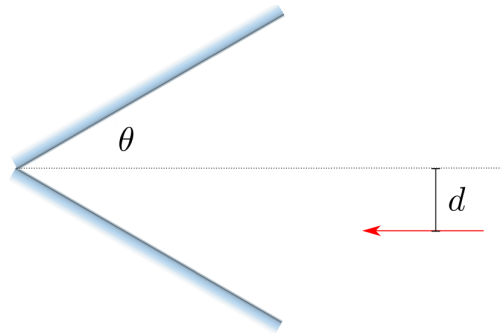   Evaluate your answer for $R = 0.5\,m$, $d = 0.4\,m$, and $\theta = \frac{\pi}{6}, \frac{\pi}{18}$.
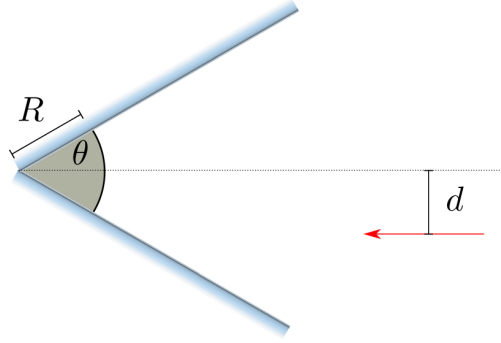
(b) Assuming we remove the detector, how many total reflections will occur? Evaluate for $\theta = \frac{\pi}{4}, \frac{\pi}{6}, \frac{\pi}{18}$. Does your answer depend on the value of d?

Hint: Calculating the incident and reflected angles at every reflection will get very tedious. Find the "image" of the photon and see where it goes.

—Pedram Amani

Optimal Illusion

When and how do mirages form?

If light tries to enter a medium with a much higher refractive index, it will sometimes *reflect* instead of refracting. This is called *total internal reflection* and it occurs when Snell's law would naively predict $\sin \theta_2 > 1$, which is impossible! More precisely, total internal reflection occurs for

$$\frac{n_2}{n_1} \sin \theta_1 > 1.$$

The index of refraction for most materials depends linearly on the density of the material. For instance, air at $25°$ (and typical air pressure) has a refractive index of 1.0003. However, when the density ρ is lower, the refractive index decreases (all the way down to 1 at vacuum). We can relate density to other properties of the gas using the *ideal gas law*:

$$P = \rho k_B T,$$

where k_B is Boltzmann's constant and $\rho = N/V$ is the number density of the gas, i.e. particles per unit volume.

Highway mirage, courtesy of Joe Orman.

1. Assuming the the index of refraction scales linearly with density, show using dimensional analysis that the refractive index of air is approximately

$$n(\rho) = 1 + 0.0003 \cdot \frac{\rho}{\rho_0},$$

where ρ_0 is the density of air at $25°$ C and standard atmospheric pressure.

2. Using the ideal gas law, show that the refractive index as a function of temperature is

$$n(T) = 1 + 0.0003 \cdot \frac{298 \text{ K}}{T}.$$

Recall that $0°$ C $= 273$ K.

3. Touring the Australian outback, I hit the brakes when I saw that the highway ahead was covered in water. I'd heard that flash-floods were common at this time of year, so I stopped well short of the water (≈ 200 meters). After waiting nearly an hour, I saw that the water was beginning to recede, so I crept my car closer to the "flood".

   The water retreated exactly as quickly as I approached. The whole affair had been a mirage in the desert! If the air outside was $25°$, how hot was the air just above the road? See figure below for a sketch of the problem.

—Daniel Korchinski