UBC Physics Circle

Session 5: Problems

December 5, 2019

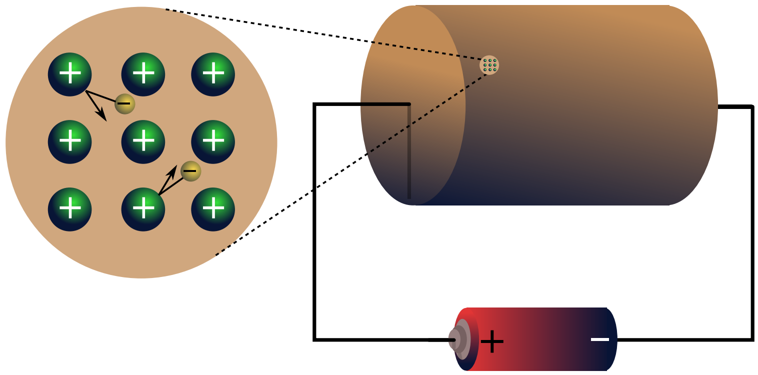
Drude, Where's My Car?

How can a constant force lead to constant velocity?

1. You are driving your car through a school zone, where the posted speed limit is 30 kph. As a safety precaution, speed bumps have been installed every ℓ metres. Because you are an edgy driver, you drive through the school zone as follows: when you reach the speed bump, you essentially instantaneously decelerate to 0 kph (so as to avoid hitting the speed bump quickly), then accelerate at 2 m/s^2 until you hit the next speed bump, where you again instantaneously decelerate to 0 kph, and so on.

   (a) What is the time elapsed between collisions with the speed bumps? What is your maximum speed in the school zone? What about your average speed? You should leave your answer in terms of the parameter ℓ.

   (b) How close together should the speed bumps be installed to ensure that your maximum speed is under the speed limit?



2. There is a simple (i.e. wrong) model for the conduction of electricity through a conductor, known as the *Drude model* (pronounced *drew-duh*). In this model, the conductor consists of ions (atomic nuclei + tightly bound electrons) and conduction electrons of mass m_e. The ions are heavy and remain fixed in place, while the conduction electrons are essentially free to move throughout the solid. We will use this model to understand the conduction of electricity in a wire of length L and cross-sectional area A subject to a voltage difference V between its endpoints.

   (a) What is the force exerted on an individual electron in the wire due to the potential difference V? You can assume that the force is constant.

   Hint. Recall that a potential different V imparts energy V per unit charge. Work (force times distance) is a form of energy.

   (b) In the Drude model, we treat the conduction electrons as if they move with a constant *drift velocity* v_{drift}. Why does this appear inconsistent with Newton's second law, at first sight? And what do you need to assume about electrons in the wire for (roughly) constant drift velocity to be reasonable?

   Hint. What is the analogy to part 1(a)?

   (c) Suppose the conduction electrons collide with atoms, with average time τ between collisions. Estimate the drift velocity v_{drift}, again using the analogy to part (a).

(d) Consider a cross-section of the wire, a surface with area A. The current I through the wire is the amount of charge passing through this surface per unit time. Express I in terms of the density n of conduction electrons in the material, the dimensions of the wire A and L, and other constant parameters.

(e) Recall *Ohm's law*, $V = IR$. Using this and the results of the previous question, what is the resistance R of the wire?

3. (a) Copper is an excellent conductor, frequently used for making wires. A chunk of copper has density $\rho = 8.92 \times 10^3$ kg/m^3, and the atomic mass of copper is 63.5 (i.e. 63.5 grams per mol). It also has 1 conduction electron per ion. The resistance of a copper wire with length 10 cm and cross-sectional radius 0.5 mm is measured to be $2 \times 10^{-3}\ \Omega$. Assuming that we apply a voltage 1 V to this wire, find the average time between electron collisions.

(b) Using the values from the previous part, determine the drift velocity if we apply a voltage difference 1 V. Is it larger or smaller than you expected? If I walk into a dark room and flip the light switch, how does the lightbulb illuminate so quickly?

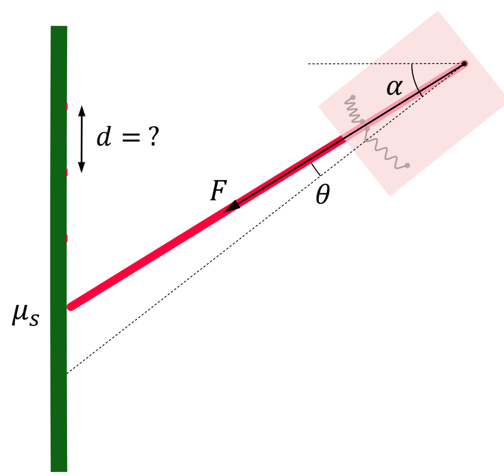
— Chris Waddell

Leaping Limestone

Why can you draw dotted lines on a chalkboard?

Hold a piece of chalk firmly at one end and place the other on the chalkboard. Angle it away from the direction of movement, and while applying slight pressure, start dragging it down. After some practice, you can easily draw dotted lines. If you haven't given it a go, try now! We are going to explore a simple mechanical model of this system, and try to derive the dot spacing (distance between adjacent dots).

1. Before looking at our proposed model however, you should try to devise your own. This is a good way to practice being a real physicist!
   (a) List the physical parameters that define this problem (e.g. length of chalk).
   (b) Try varying some of these parameters (for instance, setting some to zero) while keeping others unchanged. How is the dot spacing affected?
   (c) Try to come up with a physical model that is consistent with your observations.
2. In our model, a piece of red chalk, length ℓ, is held in your hand, represented below by the beige box. Initially, the chalk is held straight, such that the angle $\theta = 0$ and the chalk makes an angle α with the horizontal. You place the chalk on the board, exerting force F along its length, and start moving your hand down without changing its orientation. The coefficient of static friction between the two is μ_s.



We model your grab as a torque τ applied to the chalk about its held end:

$$\tau = M\theta$$

where M is a constant with units of torque. Assuming $\theta \ll \alpha$ and that you move your hand slowly, find the angle θ_0 at which the chalk slips and hence the dot spacing d.

— Pedram Amani