UBC Physics Circle

Session 4: Solutions

November 21, 2019

Exoplanet Transit

1. (a) The fractional flux decrease is equal to the fractional area decrease; this is the ratio of planet to star cross-sectional area:

$$\frac{\Delta F}{F} = \frac{\pi R_{\text{planet}}^2}{\pi R_{\text{star}}^2}.$$

Reading the value at the bottom of the dip to be $F_{\text{min}} \approx 0.9872$, $F = 1$, we get

$$R_{\text{star}} \approx 1.84 \cdot 7 \times 10^8\,\text{m}\sqrt{1 - 0.9872} \approx 1.457 \times 10^8\,\text{m}.$$

This is approximately twice the radius of Jupiter.

(b) This answer can be read directly off the plot. It is the distance between two dips, approximately 4.1 days.

2. (a) If the potential energy $U = GMm/a$ and kinetic energy $K = mv^2/2$ are equated, we find the velocity is

$$v^2 = \frac{2GM}{a}.$$

(b) The period is the circumference of orbit, $2\pi a$, divided by the velocity, so $T = 2\pi a/v$. Substituting our previous answer, we find

$$T^2 = \frac{2\pi^2 a^3}{GM}$$

Note. This is slightly wrong. The expression should have a factor of 4 rather than 2 in the numerator, due to subtleties in the correct kinetic energy. This is not relevant to an order of magnitude estimate!

(c) Inverting the equation from the previous question, we get

$$a = \left(\frac{GMT^2}{2\pi^2}\right)^{1/3}.$$

Setting $T = 4.1$ days$= 354240$ seconds, $G = 6.674 \times 10^{-11}$ kg^{-1}m^{3}s^{-2} and $M = 1.47 \cdot 2 \times 10^{30}$ kg, we find $a = 1.076 \times 10^{10}$ m.

3. (a) The radius is $2.2\,R_J$.
   (b) This is 0.07 AU, i.e. 0.07 times the distance between the Earth and the Sun. This is a huge planet located extremely close to its star!
   (c) Using the equation from Problem 1(a), we find $\Delta\, F = R_\oplus^2 / R_{\text{star}}^2 \approx 2.5 \times 10^{-5}$. This is minuscule!

Spectroscopic Binaries

1. (a) Two sets of displaced spectral lines will be present (assuming both radial velocities are nonzero at the time of the snapshot).
   (b) They will alternate being redshifted/blueshifted.

2. (a) If a stationary source emits light waves at a frequency f, and we are standing a distance x away from the source, the wave will take a time $t = x/c$ to reach us. If instead the source is moving at velocity v, then from one wave emission to the next the source will move $\Delta x = v/f$. This next wave then reaches us in a time:

$$t' = \frac{x + \Delta x}{c} = \frac{x + v/f}{c}, \tag{1}$$

   which is a time difference:

$$\Delta t = t' - t = \frac{v}{cf}. \tag{2}$$

   So instead of receiving a wave every period $1/f$, we now receive a wave with period

$$\frac{1}{f'} = \frac{1}{f} + \Delta t = \frac{1}{f}\left(1 + \frac{v}{c}\right). \tag{3}$$

   Using $\lambda = c/f$, we have

$$\lambda' = \lambda\left(1 + \frac{v}{c}\right) \tag{4}$$

   or rearranging for v

$$v = c\left(\frac{\lambda'}{\lambda} - 1\right). \tag{5}$$

   (b) The wavelength is slightly longer, so it is redshifted and thus moving away from us. Using equation 5, we find

$$v = c\left(\frac{656.4}{656.3} - 1\right) \simeq 46\,\mathrm{km/s} \tag{6}$$

   (c) The star system as a whole is moving away from us at 40 km/s.

(d) The orbital velocity is found when the star is moving along the line-of-sight (i.e. maximum radial velocity subtracted from the system radial velocity). For star A this is $60 - 40 = 20$ km/s. For star B we have $140 - 40 = 100$ km/s.

(e) With circular orbits, the stars will travel $2\pi r$ in one period T. The period of the sinusoids gives us $T = 8$ days, or 691200 s. Therefore star A has an orbital radius

$$r_A = v_A T/2\pi = 20000 \cdot 691200/2\pi \simeq 2.2 \times 10^6 \,\mathrm{km} \tag{7}$$

and for star B we have

$$r_B = v_B T/2\pi = 100000 \cdot 691200/2\pi \simeq 1.1 \times 10^7 \,\mathrm{km}. \tag{8}$$

3. (a) In a gravitationally bound system (i.e. stable), the COM frame must have zero total momentum. Thus the stars at any point in time will be travelling in opposite directions, with the vector connecting them intersecting the COM (hence orbiting about the COM). Since the total momentum of the system is zero, we have

$$M_A \mathbf{v_A} + M_B \mathbf{v_B} = 0 \implies M_A|\mathbf{v_A}| = M_B|\mathbf{v_B}|. \tag{9}$$

(b) The vector connecting the two stars will rotate about the centre of mass, so we can apply Kepler's third law to this system. The "semi-major axis" is the length of this vector, $a = r_A + r_B$, or

$$a = \frac{T}{2\pi}(v_A + v_B). \tag{10}$$

Thus we have

$$\begin{aligned} \frac{a^3}{T^2} = \frac{\frac{T^3}{(2\pi)^3}(v_A + v_B)^3}{T^2} &= \frac{GM}{(2\pi)^2} \\ \frac{T}{2\pi}(v_A + v_B)^3 &= GM \\ \implies M = M_A + M_B &= \frac{T}{2\pi G}(v_A + v_B)^3. \end{aligned} \tag{11}$$

(c) We first isolate M_B in equation 9

$$M_B = M_A \frac{v_A}{v_B}. \tag{12}$$

Substituting this into equation 11 allows us to solve for M_A

$$\begin{aligned} M_A\left(1+\frac{v_A}{v_B}\right) &= \frac{T}{2\pi G}(v_A+v_B)^3 \\ M_A &= \frac{T}{2\pi G}\frac{(v_A+v_B)^3}{\left(1+\frac{v_A}{v_B}\right)} \\ M_A &= \frac{691200}{2\pi \cdot 6.67 \cdot 10^{-11}}\frac{120000^3}{1+1/5} = 2.37\times 10^{30}\,\mathrm{kg} \simeq 1.2\,\mathrm{M}_\odot. \end{aligned} \tag{13}$$

And thus M_B is

$$M_B = 1.2\,\mathrm{M}_\odot \cdot \frac{1}{5} \simeq 0.2\,\mathrm{M}_\odot. \tag{14}$$

Bonus. Since we only observe the (projected) maximum radial velocities, v_r, the actual velocity in the orbital plane can be recovered as

$$v = \frac{v_r}{\sin i}. \tag{15}$$

Substituting this into equation 11 gives us

$$M_A + M_B = \frac{T}{2\pi G}\frac{(v_A+v_B)^3}{\sin^3 i}. \tag{16}$$

from which we can compute both masses as before. However, i is impossible to know from the radial velocity curves alone, since we are measuring $v \sin i$ and know neither v nor i! For a circular orbit, the amplitudes of the radial velocity curves will be reduced as i decreases to $0°$.