# UBC Physics Circle

Session 5: Solutions

*December 5, 2019*

## Drude, Where's My Car?

1. (a) Elapsed time $\tau$ (in seconds) is given in terms of distance $\ell$ (in metres) by

$$\ell = \frac{1}{2} \times 2 \times \tau^2 \implies \boxed{\tau = \sqrt{\ell}}\,. \tag{1}$$

Maximum speed and average speed (in m/s) are

$$\boxed{v_{\text{max}} = 2 \times \sqrt{\ell}}\,, \qquad \boxed{v_{\text{avg}} = \sqrt{\ell}}\,. \tag{2}$$

Here, we used that $v_{\text{avg}} = \ell/\tau$.

(b) We note that

$$v_{\text{limit}} = (30\,\text{km/hr}) \times \left(\frac{1000\text{ m}}{1\text{ km}}\right) \times \left(\frac{1\text{ hr}}{3600\text{ s}}\right) = 8.3\text{ m/s}\,, \tag{3}$$

so in order to have $v_{\text{max}} \leq v_{\text{limit}}$, we should take

$$\boxed{\ell \leq \left(\frac{v_{\text{limit}}}{2}\right)^2 = 17.36\text{ m}}\,. \tag{4}$$

2. (a) On the one hand, force does work, imparting an energy $W = FL$. On the other hand, we are told that a potential difference $V$ imparts energy $V$ per unit charge. Since an individual electron has charge $e$ (ignoring minus signs), we have

$$eV = W = FL,$$

and hence

$$\boxed{F = \frac{eV}{L}}\,. \tag{5}$$

(b) The apparent incompatibility would be in claiming that a constant force gives rise to a constant velocity, as opposed to a constant acceleration. However, we have neglected collisions of conduction electrons with static ions; analogizing with part (a), we might expect that the electrons will periodically accelerate (due to the applied potential difference) and decelerate (due to collisions), so that one can associate a constant average velocity to the motion.

(c) We know from part (i) that the acceleration of the electron is

$$a = \frac{F}{m_e} = \frac{eV}{m_e L} \,, \tag{6}$$

so as in part (a), we can write

$$\boxed{v_{\text{drift}} \equiv v_{\text{avg}} = \frac{1}{2} v_{\max} = \frac{a\tau}{2} = \frac{eV\tau}{2m_e L}} \,. \tag{7}$$

(d) We have

$$\boxed{I = nAev_{\text{drift}} = \frac{nAe^2 V\tau}{2m_e L}} \,. \tag{8}$$

(e) Since $I = V/R$,

$$\boxed{R = \frac{2m_e L}{nAe^2\tau}} \,. \tag{9}$$

3. (a) We have conduction electron density

$$n = \frac{(6.02 \times 10^{23}\ \text{atoms/mol}) \times (8.92 \times 10^3\ \text{kg/m}^3)}{63.5 \times 10^{-3}\ \text{kg/mol}} = 8.46 \times 10^{28}\ \text{atoms/m}^3 \,. \tag{10}$$

We therefore have

$$\begin{aligned} \tau &= \frac{2m_e L}{nAe^2 R} \\ &= \frac{2 \times (9.11 \times 10^{-31}\ \text{kg}) \times (0.1\ \text{m})}{(8.46 \times 10^{28}\ \text{atom/m}^3) \times (\pi \times 0.0005^2\ \text{m}^2) \times (1.602 \times 10^{-19}\ \text{C})^2 \times (0.002\ \Omega)} \\ &\approx \boxed{5.4 \times 10^{-14}\ \text{s}} \,. \end{aligned} \tag{11}$$

(b) The drift velocity is

$$\begin{aligned} v_{\text{drift}} &= \frac{eV\tau}{2m_e L} \\ &= \frac{(1.602 \times 10^{-19}\ \text{C}) \times (1\ V) \times (5.4 \times 10^{-14}\ \text{s})}{2 \times (9.11 \times 10^{-31}\ \text{kg}) \times (0.1\ \text{m})} \\ &\approx \boxed{4.7 \times 10^{-2}\ \text{m/s}} \,. \end{aligned} \tag{12}$$

The electrons don't have to move from the switch to the bulb to illuminate the bulb; the induced electric field throughout the wire is almost instantaneous, and is responsible for the current.

*Note.* This is probably still too high a value for the drift velocity, owing to simplifications of the model.

## Leaping Limestone

1. (a) Some relevant parameters are:
    - Length of chalk $(l)$
    - Coefficient of static friction between chalk and board $(\mu_s)$
    - Coefficient of kinetic friction between chalk and board $(\mu_k)$
    - Force applied along length of chalk $(F)$
    - Angle of chalk with respect to the board $(\alpha)$
    - Torque applied by hand on the chalk (depending on the model) $(\tau)$

   (b) Here are a few interesting observations and edge cases:
    - The spacing varies with chalk length according to an approximately square relation $d \propto l^2$ (empirical)
    - For a friction-less surface $\mu_s = 0$ we get a solid line (i.e. $d = 0$)
    - For angles $\alpha$ closer $90^\circ$ it is more and more difficult to obtain a dotted line
    - The stiffer the grip on the chalk, the smaller the dot spacing

   (c) We proposed the model stated in part (2) of the problem.

2. Dragging the chalk down, slowly increases torque applied by hand until the chalk slips. A slip occurs when the applied torque surpasses the maximum torque provided by static friction. This limit is:

$$\tau_0 = \mu_s F \cos\alpha \cdot l \cos\alpha.$$

At this slipping point, we have:

$$\tau_0 = M\theta_0 \implies \theta_0 = \frac{\mu_s F l \cos^2\alpha}{M}.$$

After the slip, the chalk returns to its equilibrium position of $\theta = 0$ (assumed to be immediate) and thus the tip travels a distance $l\theta_0$. This translates to a distance $d$ on the board of:

$$d = \frac{l\theta_0}{\cos\alpha} = \frac{\mu_s F l^2 \cos\alpha}{M}.$$