UBC Virtual Physics Circle
The Hacker's Guide to Physics

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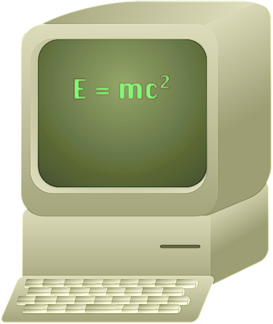
Overview

- Welcome to the UBC Virtual Physics Circle!
- Next few meetings: The Hacker's Guide to Physics.



- Don't worry. We'll be only be breaking physical laws!

What is hacking?

- Hacking can refer to breaking security systems.
- There is another meaning! Back in the day, it meant a cheeky, playful approach to technical matters.

- Example: MIT student pranks!

What is a hack?

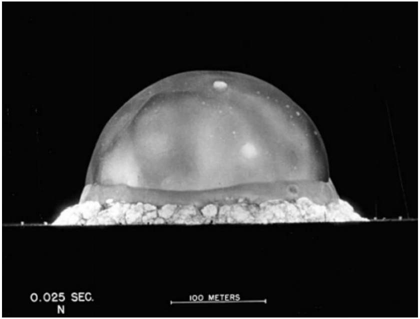
- A hack means using a technique in an ingenious way.

> *[Hackers] wanted to be able to do something in a more exciting way than anyone believed possible and show 'Look how wonderful this is. I bet you didn't believe this could be done.'*
>
> Richard Stallman

- A great hack overcomes technical limitations to achieve the seemingly impossible!

Hacking physics

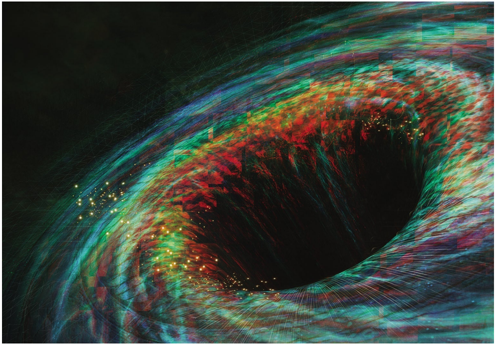
- We can hack physics with the same attitude!
- Example: the first atomic bomb test, aka the Trinity Test.



- Although the yield was classified, a physicist calculated it from the picture. This is an amazing physics hack!

Dimensional analysis

- Dimensional analysis is the ultimate physics hack: it's low-tech and applies to everything!

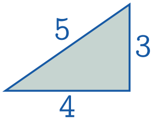


- You only need algebra and simultaneous equations.
- Not perfect, but can yield powerful results.

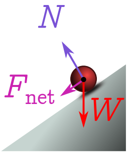
Maths vs physics

- Maths is about relationships between numbers.
- Physics is about relationships between measurements.

$$3^2+4^2=5^2$$



MATHS



PHYSICS

- A measurement tells us about some physical aspect of a system. The dimension of a measurement is that aspect!

Units and dimensions

- Measurements are packaged as numbers plus units, e.g.

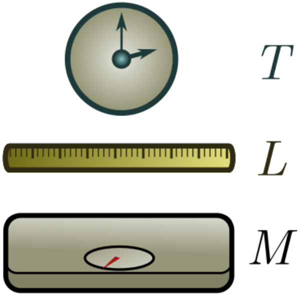
$$v = 13 \text{ m/s}, \quad E = 1.2 \times 10^4 \text{ J}, \quad t = 48 \text{ hours}.$$

- To calculate dimension: (1) throw away the number and (2) ask the unit: what do you measure?

$$\begin{aligned} [v] &= [13 \text{ m/s}] = [\text{m/s}] = \text{speed} \\ [E] &= [1.2 \times 10^4 \text{ J}] = [\text{J}] = \text{energy} \\ [t] &= [48 \text{ hours}] = [\text{hours}] = \text{time}. \end{aligned}$$

Basic dimensions

- The power of dimensional analysis comes from breaking things down into basic dimensions.
- We will use length (L), mass (M) and time (T):



- We build everything else out of these!

Algebra of dimensions

- Dimensions obey simple algebraic rules.
- Example 1 (powers):

$$[1\ \mathrm{cm}^2] = [\mathrm{cm}^2] = [\mathrm{cm}]^2 = L^2.$$

- Example 2 (different dimensions):

$$\left[4\,\frac{\mathrm{m}^3}{\mathrm{s}}\right] = \left[\frac{\mathrm{m}^3}{\mathrm{s}}\right] = \frac{[\mathrm{m}]^3}{[\mathrm{s}]} = \frac{L^3}{T}.$$

- Example 3 (formulas):

$$[F] = [ma] = [m] \times \left[\frac{v}{t}\right] = M \times \frac{L/T}{T} = \frac{ML}{T^2}.$$

Exercise 1

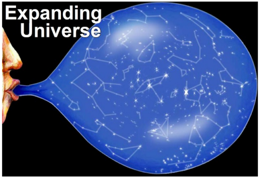
a. Find the dimensions of energy in terms of the basic dimensions L, M, T.

b. Calculate the dimension of

$$H_0 = 70\,\frac{\mathrm{km}}{\mathrm{s}\cdot\mathrm{Mpc}}$$

where $\mathrm{Mpc} = 3 \times 10^{19}$ km.

c. H_0 meaures the rate of expansion of the universe. From part (b), estimate the age of the universe.



Dimensional guesswork

- We found the dimensions of force $F = ma$, so

$$\text{physical law} \quad \Longrightarrow \quad \text{dimensions.}$$

- You can sometimes reverse the process!

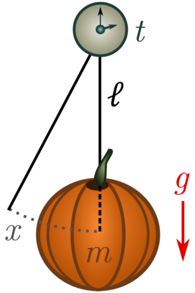
$$\text{dimensions} \quad \Longrightarrow \quad \text{physical laws.}$$

- Using these relations, you can learn other properies of a system, e.g. the age of the universe from H_0, so

$$\text{dimensions} \quad \Longrightarrow \quad \text{other physical properties.}$$

Pumpkin clock 1: setup

- The general method is easier to show than tell.
- Attach a pumpkin of mass m to a string of length ℓ and give it a small kick. It starts to oscillate.



- Our goal: find the period of oscillation, t.

Pumpkin clock 2: listing parameters

- We start by listing all the things that could be relevant:
  1. the pumpkin mass m;
  2. the string length ℓ;
  3. the size of the kick, x;
  4. gravitational acceleration, g.
- Not all the parameters are relevant!
- We can show with a few experiments that pendulums are isochronic: the period does not depend on the kick!
- Determining relevant quantities takes physics!

Pumpkin clock 3: putting it all together

- Now list dimensions for the remaining parameters:
  1. pumpkin mass $[m] = M$;
  2. string length $[\ell] = L$;
  3. finally, acceleration $[g] = [9.8 \text{ m/s}^2] = L/T^2$.
- Write the target as a product of powers of parameters:

$$t \sim m^a \ell^b g^c.$$

- Finally, take dimensions of both sides:

$$[t] = T, \quad [m^a \ell^b g^c] = \frac{M^a L^{b+c}}{T^{2c}}.$$

Pumpkin clock 4: solving for powers

$$[t] = T, \quad [m^a \ell^b g^c] = M^a L^{b+c} T^{-2c}.$$

- To find the unknown powers a, b and c, we match dimensions on the LHS and RHS:

	RHS	LHS
M	a	0
L	$b+c$	0
T	$-2c$	1

- This gives three equations for the three unknowns:

$$a = 0, \quad b + c = 0, \quad -2c = 1.$$

- This is easily solved: $a = 0, b = -c = 1/2$.

Pumpkin clock 5: pendulum period

- We now plug $a = 0, b = -c = 1/2$ into our guess:

$$t \sim m^a \ell^b g^c = m^0 \ell^{1/2} g^{-1/2} = \sqrt{\frac{\ell}{g}}.$$

- We almost got the official answer, $t = 2\pi\sqrt{\ell/g}$.
- Strengths and weaknesses:
  - $(-)$ We had to do an experiment to discard x.
  - $(+)$ We learned that m was irrelevant for free!
  - $(-)$ We missed the factor of 2π.
  - $(+)$ We're typically only off by "small" numbers!

Exercise 2

a. Instead of period t, repeat the dimensional analysis with the angular velocity $\omega = 2\pi/T$.

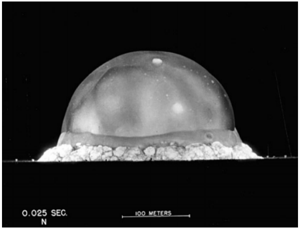
b. Show that this gives the correct result, including 2π.

c. Explain why grandfather clocks are so large.

Hint: A half period is one second.

The Trinity Test 1: parameters

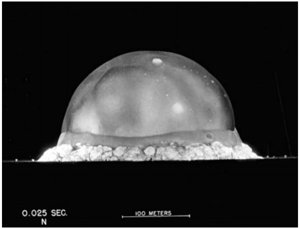
- We can now repeat G. I. Taylor's sweet hack.



- What could be relevant to the energy E released?
  - time after detonation, t;
  - radius of detonation, r;
  - mass density of air, ρ; and
  - gravitational acceleration g.
- In fact, gravity isn't relevant in an explosion like this!

The Trinity Test 1: parameters

- We can now repeat G. I. Taylor's sweet hack.



- What could be relevant to the energy E released?
  - time after detonation, t;
  - radius of detonation, R;
  - mass density of air, ρ; and
  - ~~gravitational acceleration g~~.
- In fact, gravity isn't relevant in an explosion like this!

The Trinity Test 2: putting it all together

- Find the dimensions:
  - time after detonation $[t] = T$;
  - radius of detonation $[R] = L$;
  - mass density of air $[\rho] = M/L^3$.
- Write the dimensional guess

$$E \sim t^a r^b \rho^c$$

and evaluate dimensions:

$$[E] = ML^2 T^{-2}, \quad [t^a r^b \rho^c] = T^a L^{b-3c} M^c.$$

The Trinity Test 3: solving for powers

$$[t^a R^b \rho^c] = T^a L^{b-3c} M^c, \quad [E] = T^{-2} L^2 M.$$

- Comparing powers, we have three equations:

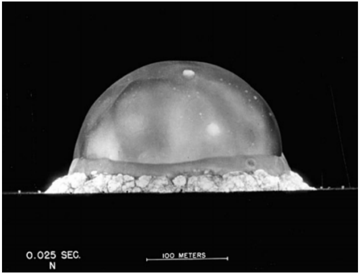
$$a = -2, \quad b - 3c = 2, \quad c = 1.$$

  Plugging the third equation into the second gives $b = 5$.
- This gives our final dimensional guess:

$$E \sim t^a R^b \rho^c = \frac{\rho R^5}{t^2}.$$

Exercise 3

a. Recall that air weighs about 1 kg per cubic meter. Use this, along with the image, to estimate E in Joules.



b. A reasonable estimate is $E \sim 10^{13}$ J. Express this in kilotons of TNT, where

$$1 \text{ kiloton of TNT} = 4.2 \times 10^{12} \text{ J}.$$

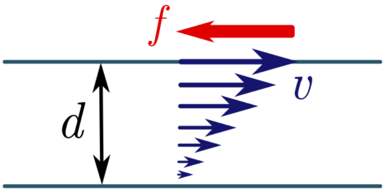
Viscosity 1: informal

- Our last example will be viscous drag on a sphere.
- Fluids have a sort of internal stickiness called viscosity.

- High viscosity fluids like honey are goopy and flow with difficulty; low viscosity fluids like water flow easily.

Viscosity 2: formal

- Formally, viscosity is resistance to forces which shear, or pull apart, nearby layers of fluid.



- Drag a plate, speed v, across the top of a fluid, depth d.
- The fluid resists with some pressure f, proportional to v and inversely proportional to d.

Exercise 4

a. Find the dimensions of pressure, $f = F/A$.

b. The viscosity of the fluid μ is defined as the constant of proportionality

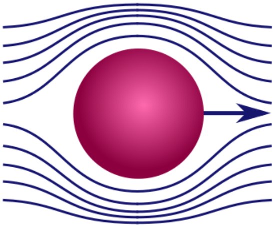
$$f = \mu \left(\frac{v}{d} \right).$$

Show that viscosity has dimensions

$$[\mu] = \frac{M}{LT}.$$

Viscous drag 1: parameters

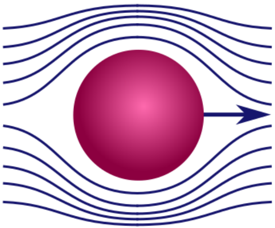
- Now imagine dragging a sphere through a viscous fluid.



- Our goal: find the drag force on the sphere! Parameters:
  - viscosity of fluid, μ;
  - radius of the sphere, R;
  - speed of the sphere, v;
  - density of fluid ρ and mass of sphere m.

Viscous drag 2: creeping flow

- If the sphere moves quickly, mass is relevant.
- If it moves slowly, it smoothly unzips layers of fluid, and mass is not important. This is called creeping flow.



- The parameters for creeping flow, with dimensions, are:
  - viscosity of fluid $[\mu] = M/LT$;
  - radius of the sphere $[R] = L$;
  - speed of the sphere, $[v] = L/T$.

Viscous drag 3: putting it all together

- Thus, we have a guess for drag force $F_{\text{drag}} \sim \mu^a R^b v^c$.
- Dimensions on the LHS and RHS are

$$[F_{\text{drag}}] = MLT^{-2}, \quad [\mu^a R^b v^c] = M^a L^{b+c-a} T^{-a-c}.$$

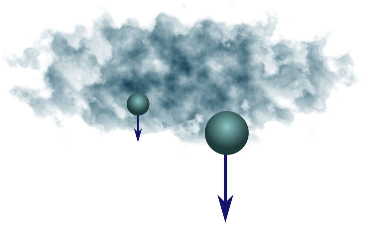
- Equating the dimensions gives

$$a = 1, \quad b + c - a = 1, \quad a + c = 2.$$

- This is clearly solved by $a = b = c = 1$.

Stokes' law

- Plugging the powers $a = b = c = 1$ in gives
$$F_{\text{drag}} \sim \mu R v.$$
- Again, we've missed a number! Stokes' law adds 6π:
$$F_{\text{drag}} = 6\pi\mu R v.$$
- This simple result has many amazing consequences. For instance, it explains why clouds float!



Exercise 5

a. Consider a spherical water droplet of radius r and density ρ, slowly falling under the influence of gravity in a fluid of viscosity μ. Show the terminal velocity is

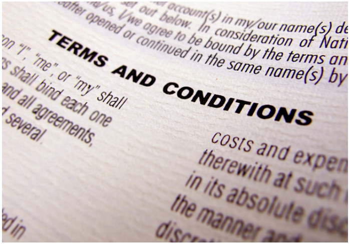
$$v_{\text{term}} = \frac{2\rho r^2 g}{9\mu}.$$

b. A typical water vapour droplet has size $r \sim 10^{-5}$ m, and cold air has viscosity $\mu \sim 2 \times 10^{-5}$ kg/m s. Find v_{term}.

c. Based on your answers, explain qualitatively why clouds float and rain falls.

Final subtleties

- Here are a few subtleties.



- Too many parameters. If parameters $>$ basic dimensions, dimensional analysis doesn't work. (Buckingham π.)
- No numbers. We can't determine numbers out the front, e.g. Stokes' 6π. Thankfully these are usually small.
- Other dimensions. There is more to physics than MLT!

Questions?

Next time: Fermi estimates!