Dimensional analysis solutions.

Problem 1. Solution.

We want to find a, b, and c s.t.

$r = KE^a \rho^b t^c$ is dimensionally consistent.

- First we write the dimensions of each term of the equation:

$[K] = 1$

$[E] = ML^2T^{-2}$

$[\rho] = ML^{-3}$

$[t] = T$

$[r] = L$

Now, we can write the following equation:

$$L = (M^a L^{2a} T^{-2a})(M^b L^{-3b})(T^c)$$

$$L = M^{a+b} L^{2a-3b} T^{-2a+c}$$

So, we can now write a system of equations:

(1) $a+b=0$ (so that $M^{a+b}=1$)

(2) $2a-3b=1$ (so $L^{2a-3b}=L$)

(3) $-2a+c=0$ (so $T^{-2a+c}=1$)

Now, we solve the system of equations.

$$\begin{cases} a+b=0 \\ 2a-3b=1 \end{cases} \qquad \begin{cases} 2a-3b=1 \\ -2a+c=0 \end{cases}$$

$$\downarrow \qquad\qquad \downarrow$$

$-2a-2b=0$

$2a-3b=1$

$-5b=1$

$b=-1/5$

$-3b+c=1$

$-3(-1/5)+c=1$

$c=1-3/5$

$c=2/5$

Finally, $a+b=0$, so $a=-b=1/5$.

Answer: $a=1/5$, $b=-1/5$, $c=2/5$.

Problem 2.

Given:

$$V_J = \frac{f}{K_J} \Rightarrow K_J = \frac{f}{V_J}$$

We want to find K_J in terms of h and e if there's a proportionality factor of 2.

- We first write the dimensions of each term in the equation. Note that V is energy per unit charge.

We will express the dimension of charge as Q.

$$[V_J] = ML^2T^{-2}Q^{-1}$$
$$[f] = T^{-1}$$
$$[e] = Q$$
$$[h] = ML^2T^{-1}$$
$$[K_J] = ?$$

from $K_J = \dfrac{f}{V_J}$

We can write the equation:

$$[k_J] = \frac{[f]}{[V_J]}$$

$$[k_J] = T^{-1}\left[M^{-1}L^{-2}T^{2}Q\right]$$

$$[k_J] = M^{-1}L^{-2}TQ$$

Now, note that $[h]^{-1} = M^{-1}L^{-2}T$

So $[k_J] = [h]^{-1}[e]$

And including the factor of 2, we find

$$k_J = \frac{2e}{h}$$

Problem 3. Solution

Given: $R_H = \frac{1}{n} R_K$.

We see that $[R_H] = [R_K]$

Then, we need to find the dimensions of resistance.

I Think that the easiest way to find the dimensions of resistance is from Ohm's Law.

$$R = \frac{V}{I} \quad \text{and} \quad I = \frac{\Delta Q}{\Delta t}$$

$$\text{So } [R] = \frac{[V]}{[I]} = [V][I]^{-1}$$

and $[I] = QT^{-1} \Rightarrow [I]^{-1} = Q^{-1}T^{1}$

and also $[V] = ML^2T^{-2}Q^{-1}$

So: $[R_K] = ML^2T^{-1}Q^{-2}$

Now, since $[h] = ML^2T^{-1}$ and $[e] = Q$, we conclude:

$$R_K = \frac{h}{e^2}$$