Solution to CAP 2022

1 Long Question

Q1

(a)

$$G\frac{mM}{R^2} = m\frac{v_1^2}{R} \quad \text{and} \quad g = G\frac{M}{R^2}$$
$$v_1^2 = gR$$
$$v_1 = \sqrt{gR} \approx 7906\text{m/s}$$

(b)

$$\frac{1}{2}mv_2^2 - \frac{GmM}{R} = 0$$
$$\frac{1}{2}v_2^2 = gR$$
$$v_2 = \sqrt{2gR} \approx 11180\text{m/s}$$

(c)

Let v' be the velocity after leaving the earth.

$$\frac{1}{2}mv_3^2 - \frac{GmM}{R} = \frac{1}{2}mv'^2$$
$$\frac{1}{2}v_3^2 - \frac{1}{2}v_2^2 = \frac{1}{2}v'^2$$
$$v_3 = \sqrt{v_2^2 + v'^2}$$

Let r be the distance between the Earth and the Sun. The speed required to escape from the Sun is given by

$$\frac{1}{2}v'^2 - \frac{GM_{sun}}{r} = 0$$

where $\frac{GM_{sun}}{r^2} = \frac{v^2}{r} \Rightarrow \frac{GM_{sun}}{r} = v^2$

Hence $\frac{1}{2}v'^2 - v^2 = 0$, which implies $v' = \sqrt{2}v$

But we can save some speed using the fact that Earth is orbiting around the Sun and we can launch the spaceship along the orbiting direction. In this way,

$$v' \to \sqrt{2}v - v = (\sqrt{2} - 1)v$$

$$\therefore v_3 = \sqrt{v_2^2 + (\sqrt{2} - 1)^2 v^2} = \sqrt{2gR + (\sqrt{2} - 1)^2 v^2}$$

To estimate the values of v_1, v_2 and v_3, we look from the Data table that

$$
\begin{aligned}
g &= 9.81 m/s^2 \\
R &= 6.371 \times 10^6 m \\
r &= 1.4958 \times 10^{11} m \\
G &= 6.67 \times 10^{-11} Nm^2/kg^2 \\
M_{sun} &= 1.99 \times 10^{30} kg
\end{aligned}
$$

Hence,

$$v_1 = \sqrt{gR} \approx 7906 \text{ m/s}$$

$$v_2 = \sqrt{2gR} \approx 11180 m/s$$

$$v_3 = \sqrt{2gR + (\sqrt{2}-1)^2 v^2} \qquad \text{where } v = \sqrt{\tfrac{GM_{sun}}{r}} \approx 29.789 \times 10^3 \text{ m/s}$$
$$\Rightarrow v_3 \approx 16651 \text{ m/s}$$

Q2

The Snell's law applies also to sound waves

We have $n_1 sin\alpha = n_2 sin\beta$ where α is incident angle (in air), β is refracted angle (in water), both measured from the line perpendicular to the surface, n_1 is refraction index of air and n_2 is refraction index of water.

$$\alpha = 90^\circ - 77^\circ = 13^\circ$$
$$\beta = 85^\circ$$

we measure it on the picture to the middle of the zone which the fish is avoiding

$$n_1/n_2 = v_{water}/v_{air} = sin\beta/sin\alpha = 4.4$$

The refraction index is inversely proportional to the speed of sound in the media

$$v_{water} = 4.4 \times 340m/s \approx 1500m/s$$

Q3

(a)

Conservation of momentum:

$$M_n U_0 = M_d V_d + M_n V_n$$
$$U_0 = 2V_d + V_n$$

Conservation of Kinetic Energy:

$$\frac{1}{2} M_n U_0^2 = \frac{1}{2} M_d V_d^2 + \frac{1}{2} M_n V_n^2$$
$$U_0^2 = 2V_d^2 + V_n^2$$

Solving from these two equations:

$$\frac{V_d}{U_0} = \frac{2}{3}$$

(b)

Initial Kinetic Energy $= \frac{1}{2} M_n U_0^2$

Deuteron Kinetic Energy:

$$\frac{1}{2} M_d V_d^2 = \frac{1}{2} 2Mn4/9U_0^2$$
$$KE_d/KE_n = 8/9 = 89\% = 0.889$$

(c)

8/9 KE_0 is gained for deuteron

1/9 KE_0 is left with neutron after each collision

After N collision,

$$(1/9)^N \times 20 \times 10^6 eV = 2 \times 10^{-2} eV$$
$$N = 9.4$$

10 collisions is required.

(d)

$$mV_0 = mV_1 \cos\theta_1$$
$$0 = mV_1 \sin\theta_1 + 2mV_2 \sin\theta_2$$
$$\frac{1}{2}mV_0^2 = \frac{1}{2}mV_1^2 + \frac{1}{2}2mV_2^2$$

Solving this set of equations would give us higher $\frac{V_1}{V_0}$ ratio compared to heads on collision, so the number of collisions would increase if collisions were at angle.

2 Short Question

1.

(C)

During the collision, the force acting on the wood is the same as the force acting on the bullet. As the change of momentum equals the force integrated over time of collision, they have to be the same.

2.

(C)

Since the whole system is static initially, the center of mass of the system should not change throughout the process.

Let v be the downward speed of the balloon

$\Rightarrow$ Tom moves at $s - v$ relative to the ground

$\Rightarrow m(s - v) - Mv = 0$

$\Rightarrow v = \frac{ms}{M+m}$

3.

(B)

After Ann threw the ball to Betty:

$$\begin{aligned} mu &= Mv_A \quad \text{where } v_A \text{ is speed of Ann} \\ (M+m)v_B &= mu \quad \text{where } v_B \text{ is speed of Betty} \end{aligned}$$

After Betty threw the ball back to Ann:

$$\begin{aligned} Mv'_B - mu &= (M+m)v_B \quad \text{where } v'_B \text{ is new speed of Betty} \\ (M+m)v'_A &= mu + Mv_A \quad \text{where } v'_A \text{ is new speed of Ann} \end{aligned}$$

Solving,

$$v'_B = \frac{2mu}{M}$$
$$v'_A = \frac{2mu}{M+m}$$

Hence,

$$v'_B - v'_A = 2mu\left(\frac{1}{m} - \frac{1}{M+m}\right) = 2mu\frac{m}{M(M+m)} = \frac{2m^2u}{M(M+m)}$$

4.

(D)

$$\begin{aligned} Ft &= mv \\ K.E. &= \frac{1}{2}mv^2 = \frac{1}{2}\frac{(mv)^2}{m} = \frac{1}{2}\frac{(Ft)^2}{m} \\ \Rightarrow K.E. &\propto \frac{1}{m} \\ \Rightarrow A : B &= 2 : 1 \end{aligned}$$

5.

(C)

$$\begin{aligned} \frac{GMm}{r^2} &= m\frac{v^2}{r} \\ K.E. &= \frac{1}{2}mv^2 = \frac{1}{2}G\frac{Mm}{r} \\ E = P.E. + K.E. &= -G\frac{Mm}{r} + \frac{1}{2}G\frac{Mm}{r} = -\frac{1}{2}G\frac{Mm}{r} \end{aligned}$$

E should be decreasing because of air resistance.

$\Rightarrow r \downarrow$

$\Rightarrow K.E. \uparrow$

$\Rightarrow v \uparrow$

But $T = \frac{2\pi r}{v} \Rightarrow T \downarrow$ as $r \downarrow, v \uparrow$

6.

(B)

$$\begin{aligned} \epsilon &= \frac{d\Phi}{dt} = B\frac{dA}{dt} \\ &= B(\ell sin60^\circ)v \\ &= \frac{\sqrt{3}}{2}B\ell v \end{aligned}$$

7.

(A)

$$\lambda = \frac{c}{f} = \frac{6}{3} = 2m$$

Phase difference=$\frac{(1m)\times(sin(\frac{\pi}{6}))}{\lambda} \times 2\pi = \frac{(1)(\frac{1}{2})}{2} 2\pi = \frac{\pi}{2}$

8.

(D)

$$\frac{V}{R} = I = nqvA$$
$$v = \frac{V}{RnqA} = 1.25 \times 10^{-3} m/s$$

9.

(D)

At the start the acceleration is g. Later the drag comes in which is the same for both but heavier ball has the higher force of gravity acting on it so its acceleration is higher.

10.

(B)

On the way down as the speed is constant Force of gravity times sin(5°) must be equal force of friction = 55.5 N
On the way up total force is 2*55.5N
Power = Fd/t = Fv = 111N*6.94m/s = 771 W   800 W

11.

(B)

The water will push in the direction opposite to the acceleration so the lighter bubble will be pushed in the direction of acceleration.

12.

(D)

All the other are technically impractical and it is a real picture.

13.

(B)

From Snell's law the angle of the direction of the beam leaving the water α (relative to vertical) will be $\sin(\alpha) = \sin(10)$*n (n=1.33 is refraction index of water)
$\alpha = 13°$ distance up the wall is 2 m divided by $\tan^{-1}(13°) = 8.4$ m

14.

(E)

From Snell's law we have like before: $\sin(\alpha) = \sin(70)$*n = 1.25

So there is no angle fulfilling this equation, we will have the total internal reflection and the beam will never go above the water: E

15.

(C)

λ = h/p
energy E is given by $\mathrm{mv}^2/2 = \mathrm{p}^2/2\mathrm{m}$
$\lambda = \mathrm{h}/\sqrt{2}\mathrm{Em}$
E = 1/2m *(h/λ)^2 = 1/(2* 1.67*10^{-27} kg)* (6.63*10^{-34}Js/0.1*10^{-9} m)^2 = 1.3*10^{-20} J

16.

(C)

notice that all R5-R4 pairs form the same voltage dividers so there is no potential difference on any R1, R2 or R3 resistors.

17.

(A)

The still weight sitting on the mattress/raft has to push down the mattress to push out it's weight of water. As steel has a much higher density it will push out much less water when it drops to the bottom.

18.

(E)

After the first filter: intensity $I_1 = \frac{L}{2}$

After the second filter: intensity $I_2 = I_1 cos^2\theta = \frac{L}{2}cos^2\theta$

After the third filter: intensity $I_3 = I_2 cos^2(90-\theta) = I_2 sin^2\theta = \frac{L}{2}cos^2\theta sin^2\theta$

19.

(A)

In the stationary frame, the upper ruler moves at $v = \sqrt{3}c/2$, so $\gamma = \frac{1}{\sqrt{1-(\frac{v}{c})^2}} = 2$.

Proper length of the upper ruler= $\gamma L = 2L$

In the rest frame of upper ruler, length of lower ruler= $\frac{L}{\gamma} = \frac{L}{2}$

Time taken for the lower ruler to move from the right end to the left end as seen in the frame of upper ruler is

$$\frac{(2L-\frac{L}{2})}{-\frac{\sqrt{3}c}{2}} = \frac{\frac{3L}{2}}{\frac{\sqrt{3}c}{2}} = \sqrt{3}\frac{L}{c}$$

Since the lower ruler move from the right to the left in the frame of the upper ruler, the time on the clock should be negative, i.e. $-\sqrt{3}\frac{L}{c}$

20.

(B)

The optimal solution is to 'floor the gas' for the first half and 'floor the brake' for the second half. The first half takes time $\sqrt{(\Delta x/a)}$, and the second half takes the same amount of time by symmetry, giving total time $2\sqrt{(\Delta x/a)}$.

21.

(B)

Cutting a spring in half doubles its spring constant; if a force F would elongate the original spring by an amount x, it elongates the new spring by an amount x' = x/2 , and using F=-kx gives k' = 2k. The new system thus consists of two springs, each of whose spring constant is twice that of the original spring; the effective spring constant has thus increased by a factor of four. Since the angular frequency goes as the square root of the spring constant, the angular frequency increases by a factor of two.

22.

(B)

F_a = Centripetal force F_c = Frictional force of the road, equals to force exerted by tires exert in the backward direction so that the car moves in the forward direction (Newton's third law Action-Reaction). The horizontal force on the car's tires, $F_a + F_c = F_b$

23.

(C)

u = pc^2/E = 5 MeV/c* c^2 / 10 MeV = 0.5 c

24.

(E)

No, because it would violate the second law of thermodynamics.

25.

(B)

The acceleration tangent to the incline of the helix is given by $a_t = gsin\theta$, where θ is the angle of the incline of the helix. As it accelerates, it moves faster, according to $v = a_t t$. Since this is motion in a circle, then there is a radial acceleration given by $a_r = v^2/r = (a_t t.)^2/r$ The magnitude of the acceleration is then: $a = \sqrt{a_t^2 + a_r^2}$