UBC Physics Circle

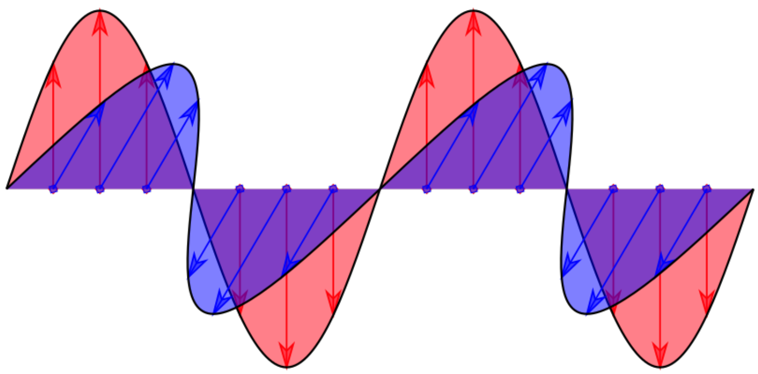
Session 1: Solutions

November 1, 2023

Linear Polarizer Sequence

I make light less intense, but the more of me you add,
the more light gets past my fence. What am I?

1. One way to view light is as a wave that has electric (red) and magnetic (blue) components. We say that the **polarization** of light is the direction that the electric component of the wave oscillates in. In the below diagram, the polarization of the light is up/down.



A **linear polarizer** is a device, such as your sunglasses, that only permits light polarized along its axis to pass through it. For example, if vertically polarized light passes through a linear polarizer whose axis is also vertical, then all of the light will pass through. If we rotate the linear polarizer clockwise $90°$ so that it's horizontal, none of the light will pass through. From here, if we rotate the linear polarizer counterclockwise $45°$, then 50% of the light will make it through, meaning that the light will be half as intense after the polarizer. The exact relationship of the light's intensity after it passes through a linear polarizer is given by **Malus' Law**:

$$I = I_0 \cos\theta^2$$

Where I_0 is the intensity of the light prior to the polarizer, I is the intensity after, and θ is the angle measured between the polarization of the light, and the axis of the linear polarizer. After the light passes through the filter, whatever light remains will have a new polarization equivalent to the axis of the polarization filter it just passed through.

(a) Vertically polarized light, with initial intensity I_0, passes through a horizontal linear polarizer. Use Malus' Law to show that the intensity of the light after the filter is 0

Solution.

$$I_1 = I_0 \cos^2 90 = 0$$

(b) Vertically polarized light, with initial intensity I_0, passes through a linear polarizer that's rotated $45°$ clockwise, and a second linear polarizer that's rotated another $45°$ from the first polarizer, so that the second one is horizontal. What is the intensity of the light after the last filter? Did adding a second filter increase or decrease the final intensity of light?

Solution. The intensity after the first filter will be:

$$I_1 = I_0 \cos^2 45°$$

The intensity after the second filter will be:

$$I_2 = I_1 \cos^2 45° = (I_0 \cos^2 45°) \cos^2 45° = I_0 \cos^4 45° = \frac{1}{4}$$

The intensity has increased!

(c) (*challenge*) Vertically polarized light, with initial intensity I_0, passes through n linear polarizers. The angle between all polarizers are equal, and arranged such that the last polarizer will be placed horizontally. Find the intensity of the light after the last filter I_n.

(hint): part (a) is the solution when $n = 1$, part (b) is the solution when $n = 2$. Can you identify the pattern and extend it to find I_n?

Solution. The angle between each successive linear polarizers' axes are $\frac{90°}{n}$ After the first filter the intensity will be:

$$I_1 = I_0 \cos^2 \frac{90°}{n}$$

After the second filter the intensity will be:

$$I_2 = I_1 \cos^2 \frac{90°}{n} = I_0 \cos^4 \frac{90°}{n}$$

Recognizing the pattern, we see:

$$I_n = I_0 \cos^{2n} \left(\frac{90°}{n} \right)$$

This sequence is increasing. The value of I_n as n approaches infinity is I_0. We can see this as we increase n, the argument inside the cosine tends closer to 0, and $\cos 0 = 1$, and 1^k for any k is 1.

2. Another way to view light is as a particle, called a **photon**. In the previous question, we considered how light behaves when we bombard many rays (or equivalently, photons) of light through linear polarizers, and saw that some portion of it gets filtered and does not pass through, according to Malus' law. Now let's consider each photon one at a time. Is it possible to know whether any particular photon will pass through the linear polarizer ahead of time? How does the universe decide which photons to let through a polarizer? Does the decision to let a photon through a polarizer get made the instant it happens, or is it made ahead of time, and we just need to observe the outcome to learn the answer for ourselves? This is exactly the question that **Bell's Theorem** answers, and it has shocking implications about the universe we live in that challenge the most basic assumptions we have.

   Let's start off by assuming that each photon has already decided whether or not it will pass through a given filter in advance. You may think this to be odd, but the universe already does this in many instances! For example, an electron's mass is already decided before we measure it. This general idea is known as **realism**.

   Continuing with the assumption, let's collect 100 photons. Let's set up three linear polarizers like the following: the first one A is vertical, the second one B is rotated $22.5°$ clockwise, and the third one C is rotated an additional $22.5°$. According to our assumption, each photon has already decided whether it will pass each polarizing filter, so we will select the 100 photons such that they all pass the first filter. We can use Malus' Law like in question 1 to determine how the experiment will behave, but for efficiency, the following are the results of this experiment:

   - All 100 photons pass A
   - 15 photons are blocked by B
   - an additional 15 photons are blocked by C

   If we removed the second filter B, the following are the results of this new experiment:

   - All 100 photons pass A
   - 50 photons are blocked by C C

   Let's introduce the function $N(+A, +B, +C)$ to count the number of photons that pass through A, B and C. $N(+A, -B)$ would count the number of photons that pass through A but do not pass through B. $N(-A)$ would count the number of photons that do not pass through A.

   (a) What is $N(+A, -C)$ using the above experiment results, when filter B is removed?

   Solution. 50 photons make it past A but are blocked by B

(b) Now, let's bring the filter B back. From part (a), we know $N(+A, -C)$, but how do these photons behave with B back in? There are two options, either we have $N(+A, +B, -C)$ or $N(+A, -B)$. Compute these two from the experimental values. Then, argue that $N(+A, -C) \leq N(+A, +B, -C) + N(+A, -B)$, and evaluate the inequality. Is the inequality obeyed?

Solution. From the experimental values $N(+A, +B, -C) = 15$, and $N(+A, -B) = 15$.

Now to show $N(+A, -C) \leq N(+A, +B, -C) + N(+A, -B)$. We're interested in the photons that pass A and don't pass C. When we introduce B, we have two varieties of photons fitting this observation of $N(+A, -C)$. The first one is the kind that passes through B and doesn't pass through C, given by $N(+A, +B, -C)$. The second one is the kind that does not pass through B, so it doesn't have a chance to pass or get blocked by C. Therefore, $N(+A, -C)$ must be less than or equal to the sum of these two varieties of photons.

But, when we calculate this inequality: $50 \leq 15 + 15 = 30$ which is not true. So the inequality is not upheld.