UBC Physics Circle

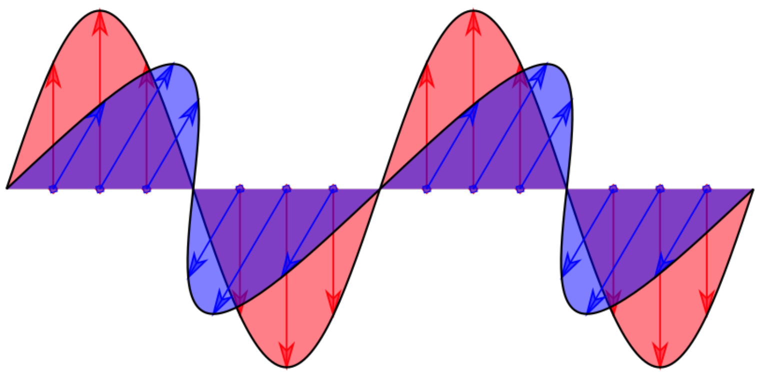
Session 1: Problems

November 1, 2023

Linear Polarizer Sequence

I make light less intense, but the more of me you add,
the more light gets past my fence. What am I?

1. One way to view light is as a wave that has electric (red) and magnetic (blue) components. We say that the **polarization** of light is the direction that the electric component of the wave oscillates in. In the below diagram, the polarization of the light is up/down.



A **linear polarizing filter** is a device, such as your sunglasses, that only permits light polarized along its axis to pass through it. For example, if vertically polarized light passes through a linear polarizer whose axis is also vertical, then all of the light will pass through. If we rotate the linear polarizer clockwise $90°$ so that it's horizontal, none of the light will pass through. From here, if we rotate the linear polarizer counterclockwise $45°$, then 50% of the light will make it through, meaning that the light will be half as intense after the polarizer. The exact relationship of the light's intensity after it passes through a linear polarizer is given by **Malus' Law**:

$$I = I_0 \cos\theta^2$$

Where I_0 is the intensity of the light prior to the polarizer, I is the intensity after, and θ is the angle measured between the polarization of the light and the axis of the linear polarizer. After the light passes through the filter, whatever light remains will have a new polarization equivalent to the axis of the polarization filter it just passed through.

(a) Vertically polarized light, with initial intensity I_0, passes through a horizontal linear polarizer. Use Malus' Law to show that the intensity of the light after the filter $I_1 = 0$.

(b) Vertically polarized light, with initial intensity I_0, passes through a linear polarizer that's rotated $45°$ clockwise, and a second linear polarizer that's rotated another $45°$ from the first polarizer, so that the second one is horizontal. What is the intensity of the light after the last filter, I_2? Did adding a second filter increase or decrease the final intensity of light?

(c) (*challenge: do question 2 first*) Vertically polarized light, with initial intensity I_0, passes through n linear polarizers. The angle between all polarizers are equal, and arranged such that the last polarizer will be placed horizontally. Find the intensity of the light after the last filter I_n. As we increase n, does I_n increase or decrease? What value does it approach as n approaches infinity?

(hint): part (a) is the solution when $n = 1$, part (b) is the solution when $n = 2$. Can you identify the pattern and extend it to find I_n?

Bell's Theorem

Your senses have lied to you. The universe is not what it seems.

2. Another way to view light is as a particle, called a **photon**. In the previous question, we considered how light behaves when we bombard many rays (or equivalently, photons) of light through linear polarizers, and saw that some portion of it gets filtered and does not pass through, according to Malus' law. Now let's consider each photon one at a time. Is it possible to know whether any particular photon will pass through the linear polarizer ahead of time? How does the universe decide which photons to let through a polarizer? Does the decision to let a photon through a polarizer get made the instant it happens, or is it made ahead of time, and we just need to observe the outcome to learn the answer for ourselves? This is exactly the question that **Bell's Theorem** answers, and it has shocking implications about the universe we live in that challenge the most basic assumptions we have.

   Let's start off by assuming that each photon has already decided whether or not it will pass through a given filter in advance. You may think this to be odd, but the universe already does this in many instances! For example, an electron's mass is already decided before we measure it. This general idea is known as **realism**.

   Continuing with the assumption, let's collect 100 photons. Let's set up three linear polarizers like the following: the first one A is vertical, the second one B is rotated $22.5°$ clockwise, and the third one C is rotated an additional $22.5°$. According to our assumption, each photon has already decided whether it will pass each polarizing filter, so we will select the 100 photons such that they all pass the first filter. We can use Malus' Law like in question 1 to determine how the experiment will behave, but for efficiency, the following are the results of this experiment:

   - All 100 photons pass A
   - 15 photons are blocked by B
   - an additional 15 photons are blocked by C

   If we removed the second filter B, the following are the results of this new experiment:

   - All 100 photons pass A
   - 50 photons are blocked by C

   Let's introduce the function $N(+A, +B, +C)$ to count the number of photons that pass through A, B and C. $N(+A, -B)$ would count the number of photons that pass through A but do not pass through B. $N(-A)$ would count the number of photons that do not pass through A.

   (a) What is $N(+A, -C)$ using the above experiment results, when filter B is removed?

(b) Now, let's bring the filter B back. From part (a), we know $N(+A, -C)$, but how do these photons behave with B back in? There are two options, either we have $N(+A, +B, -C)$ or $N(+A, -B)$. Compute these two. Then, argue that $N(+A, -C) \leq N(+A, +B, -C) + N(+A, -B)$, and evaluate the inequality. Is the inequality obeyed?

In the previous part, we found that the inequality is not obeyed. So, we have contradicted our original assumption that each photon decides ahead of time whether it passes a given polarizing filter. Can we say then, that each photon does not make this decision ahead of time? Not quite yet, because our logic has a loophole. We assumed that the photon's decision does not change after it passes a filter. What if the act of passing a filter changes the photon's decision to pass the other filters? Have we reached a dead-end?

No! Smart physicists have set up the same experiment so that a pair of **entangled** photons pass through polarizing filters at different points in space at the same time. By entanglement of a pair of photons, what we mean is they behave identically when put through the same conditions, regardless of how far apart they are separated. So, if one photon passes through a vertically oriented linear polarizer, the other entangled photon will also pass through another vertically oriented linear polarizer. What physicists did is entangle two photons, and send them through the same setup of linear polarizing filters, and tabulate the exact same experimental results as we did! Since the entangled photons pass through their respective filters simultaneously, the only way for their decision about passing a filter could change is if the two photons were communicating faster than light. This idea is known as **non-locality**.

The universe behaving locally (information travels no quicker than the speed of light) and with realism (particles in the universe have definite properties like mass, charge, position) are very natural things to assume. But, this contradiction, known as Bell's Theorem, tells us that the universe cannot preserve both $locality$ and $realism$, as we once assumed. It doesn't tell mechanism by which the universe breaks this, but tells us with certainty that the universe cannot be locally real.

Inspiration: https://youtu.be/zcqZHYo7ONs?si=9CI7J4WJjExZ9bxi — Sean Ghaeli