Anti-matter Propulsion

One proposed form of rocket propulsion for space travel is anti-matter propulsion. For reasons that we'll discuss, the most feasible form of this is antiproton-proton annihilation.

To build up to asking you to discuss the pros and cons of anti-matter engines, we'll first go over the basic physics of particle interactions and do some quick calculations.

Mass-energy equivalence ($E = mc^2$)

All objects with mass have a corresponding intrinsic energy, $E = m_0 c^2$, called the rest-mass energy.

For a moving object, its total energy (including the rest mass energy) can be written as

$$E = m_0 c^2 + \frac{1}{2} m_0 v^2$$

Question 1:

What is the rest mass energy of one proton? How about one anti-proton?

Note: you can approximate the mass of the proton as $m_p = 1.67 \times 10^{-27}$ *kg*

Relativity & the Lorentz factor

For an object moving at speed v, where v is a significant fraction of the speed of light, an object has total momentum:

$$p = \gamma m_0 v,$$

And total energy:

$$E = \gamma m_0 c^2$$

Where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

This is the total energy term including the kinetic energy of the object. Since the rest mass is $E = m_0 c^2$, the kinetic energy of a relativistic object is $E_k = (\gamma - 1)m_0 c^2$

Question 2:

What is the total energy and the kinetic energy of a proton travelling at speed 0.6c?

Question 3:

The proton-antiproton annihilation process has been proposed as an efficient a way to fuel rockets to make interstellar travel more feasible. A proton and antiproton annihilate to produce energy and unstable particles.

Some tables and values that may be useful for the problems:

Rest mass of $p^{\pm} = 938 MeV/c^2$

Rest mass of $e^{\pm} = 0.511 MeV/c^2$

Rest mass of $\pi^{\pm} = 140 MeV/c^2$

Rest mass of $\pi^{0} = 135 MeV/c^2$

Rest mass of $\mu^{\pm} = 106 MeV/c^2$

$p+\bar{p} \rightarrow n\pi^+ + n\pi^- + m\pi^0$	n≈1.5, m≈2
$\pi^0 \xrightarrow{2.2\times10^{-16}s} 2\gamma$	$E_\gamma \simeq 130\text{-}300$ MeV
$\pi^+ \xrightarrow{7.0\times10^{-8}s} \mu^+ + \nu_\mu$	$\bar{KE}(\pi^\pm) \simeq 250$ MeV
$\pi^- \xrightarrow{7.0\times10^{-8}s} \mu^- + \bar{\nu_\mu}$	$\bar{KE}(\mu^\pm) \simeq 192.3$ MeV
$\mu^+ \xrightarrow{6.2\times10^{-6}s} e^+ + \nu_e + \bar{\nu_\mu}$	$\bar{KE}(e^+) \simeq 100$ MeV
$\mu^- \xrightarrow{6.2\times10^{-6}s} e^- + \nu_\mu + \bar{\nu_e}$	$\bar{KE}(e^-) \simeq 100$ MeV
$e^- + e^+ \rightarrow 2\gamma$	$E_\gamma = 0.511$ MeV

Table 1: Proton-Antiproton Annihilation Scheme.

Figure from LaPointe, 1989 (NASA)

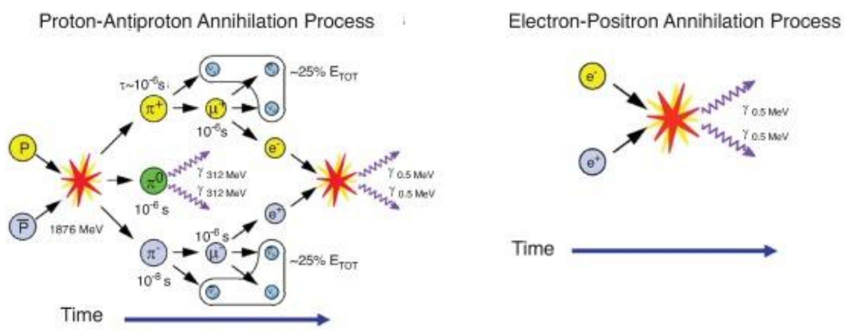

Figure from Matloff & Garrish, 2023

The first step of the proton-antiproton interaction is:

$$p^+ + p^- \rightarrow 2\pi^0 + 1.5\pi^+ + 1.5\pi^-$$

The π^0 will almost instantaneously decays into 2 γ-rays according to

$$\pi^0 \rightarrow 2\gamma$$

If the gamma rays each carry 312MeV, what is the total energy of the charged pions (π)? What is their kinetic energy? You can leave answers in MeV/c^2

The charged pions will decay in about 26 ns according to:

$$1.5\pi^+ + 1.5\pi^- \rightarrow 1.5\mu^+ + 1.5\mu^- + 1.5\nu_\mu + 1.5\,\overline{\nu_\mu}$$

Assume that the neutrinos carry a total of 156MeV out of the system.

What is the total energy of the charged muons (μ)? What is their kinetic energy?

The charged muons will decay into electrons and anti-electrons in about 6μs according to:

$$1.5\mu^+ + 1.5\mu^- \rightarrow 1.5e^+ + 1.5e^- + 1.5\nu_e + 1.5\,\overline{\nu_e} + 1.5\nu_\mu + 1.5\,\overline{\nu_\mu}$$

Assume that the neutrinos collectively carry 313MeV out of the system.

What is the total energy of the electrons/anti-electrons (e)? What is their kinetic energy? What is the fraction of the initial energy left in the engine after this step?

Question 4:

What makes the proton-antiproton annihilation a better contender for rocket propulsion that the electron-positron annihilation?

The electron-positron annihilation is:

$$e^- + e^+ \rightarrow 2\gamma$$

How might you use this proton-antiproton interaction to propel a rocket?

There are lots of different answers to this, feel free to look things up or try to come up with your own idea.

What might make an anti-matter engine difficult to implement?

Again, there are lots of answers! Please discuss as much as you can.