Bohr Model, Hydrogen Atom and Aharnov-Bohm Effect

The Aharonov–Bohm effect, is a quantum-mechanical phenomenon in which an electrically charged particle is affected by an electromagnetic potential despite being confined to a region in which both the magnetic field and electric field are zero. While we do not really have to worry about the details of this, today we can work out an order of magnitude estimation of how magnetic field may affect electrons (and thus possibly an electron in a Hydrogen Atom).

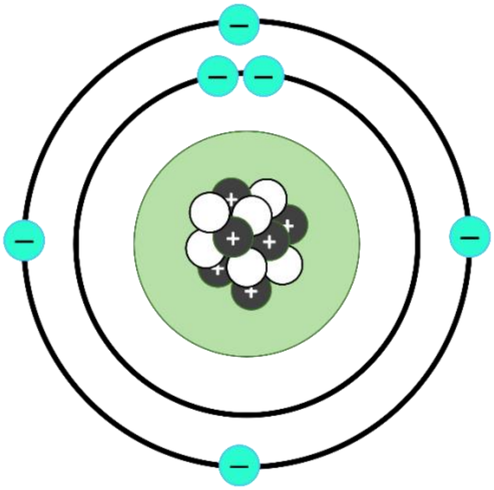
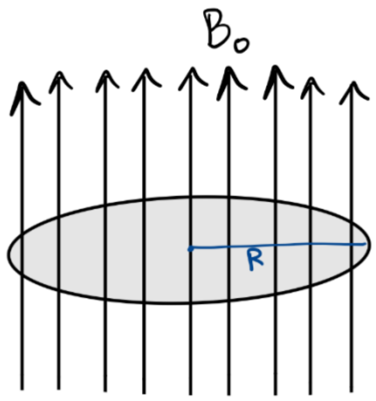
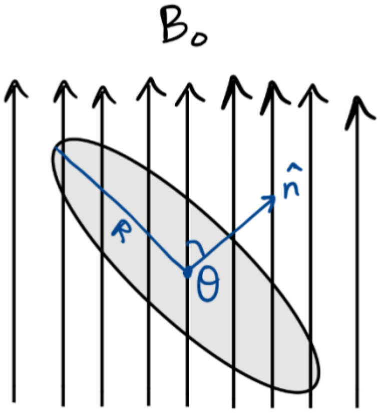


Fig (from Wikipedia): Shows the Bohr Atomic Model. The electrons orbit the nucleus in a well-defined circular path.

While this model is not accurate (can you think what might be wrong with this?), it is what we will work with for today’s estimations. For a Hydrogen Atom, we will only have one proton in the nucleus and one electron circling the proton at the Bohr radius.
The Bohr radius is $5.2 \times 10^{-11} m$.

To understand this calculation, we first need to look at magnetic flux. Magnetic flux is a measurement of the total magnetic field which passes through a given area. Flux, Φ, is the product of the magnetic field and area through which we want to know the magnetic flux.





In this figure, we can see that the magnetic field and area of shaded loop is same in both figures, yet in the right panel, the number of magnetic field lines going through the shaded loop is lower.

QUESTION 1:
Try and find an expression relating the magnetic field, magnetic flux, the angle theta and the radius of the shaded loop, R.

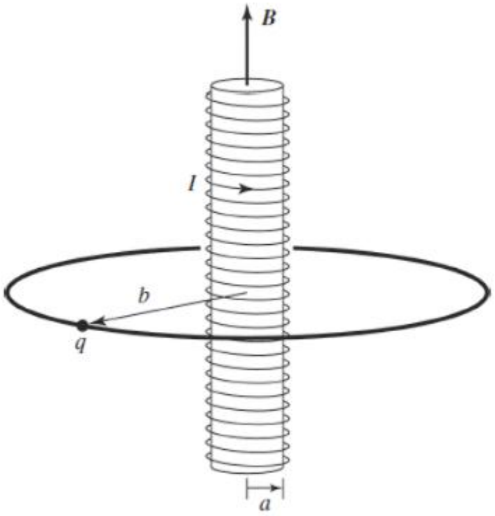
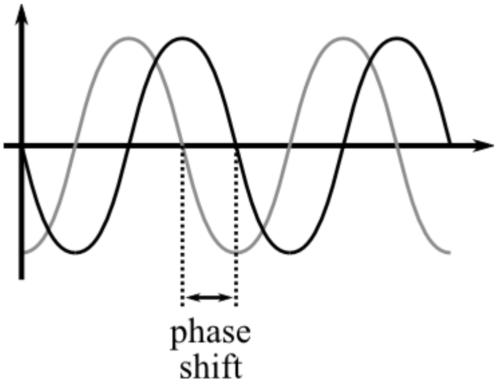


Fig 2 (From Griffiths Introduction to Quantum Mechanics): We are looking at a magnetic field being produced by a solenoid, and an electron moving in a circular path around it, defined by a radius b.

When an electron moves around a magnetic field, it accumulates a phase in its wavefunction. This phase can somewhat be thought of the phase of any other waves you are familiar with.



The phase accumulated by a electron wavefunction in a magnetic field is $\Delta\varphi = \frac{q\phi}{\hbar}$. Where ϕ is the Magnetic Flux, the reduced Planck's constant $\hbar = 1.05 \times 10^{-34} Js$ and charge of an electron, $q = 1.6 \times 10^{-19} C$.

QUESTION:
To observe a significant shift in the phase of the electron, we want $\Delta\varphi$ to be high, (let us assume we need $\Delta\varphi \approx \pi$). Estimate how much magnetic flux is required for this phase difference.

Given we know the Bohr radius of an electron (i.e, the average radius of orbit of the electron around nucleus), calculate the Magnetic field required to achieve such a magnetic flux through the loop created by an electron moving around a nucleus in Hydrogen. (Remember Bohr radius is $5.2 \times 10^{-11} m$)

Optional: The energy of electrons can be written as:

$$E_n = \frac{\hbar^2}{2m_e r_{Bohr}^2}\left(n - \frac{q\Phi}{2\pi\hbar}\right)^2, (n = \pm 1, \pm 2 \dots.)$$

For the Bohr model approximation. Here m_e is the electron mass and r_{Bohr}^2 is the square of the Bohr radius. Let us just consider the case of n=1 case.

QUESTION:

The $\Phi = 0$ case is the case where there is no magnetic field, and thus the "normal" case. Can you argue what the magnetic flux Φ, needs to be in order to observe a reasonable shift in energy of the electrons compared to the "normal" case? Is it the similar to what you calculated before?

Assuming n=1 and no magnetic field, we get the energy of an electron (in lowest energy state) is given by the equation by the Bohr Atomic Model:

$$E_1 = \frac{\hbar^2}{2m_e r_{Bohr}^2}$$

Experimentally, it has been observed that the energy of the lowest-energy electron in Hydrogen is:

$$2.2 \times 10^{-18} Joules$$

(or 13.6eV). Does the Bohr model assumption with n=1 give a similar energy? Is the Bohr model a good Atomic model?