RADIO FOREGROUNDS

STUDYING THE GALAXY TO REMOVE IT

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Talk summary

- The CMB is the echo of the big bang, and is seen as a 2.7 K blackbody. This blackbody has anisotropies (or variations) which can tell us about the contents of the universe, and its formation.
- CMB anisotropies are largely dominated by emission from the Milky way on the night sky - which we need to subtract in order to leave a map of the CMB.
- The Milky Way's radio emission can be though of as the sum four key components;

  Synchrotron emission from high-energy electrons spiralling round magnetic field lines. There are two components of synchrotron; a diffuse background which comes from cosmic ray electrons in our Galaxy's disk, and compact objects which are mainly supernova remnants (SNR).

  Free-free emission from ionized hydrogen in the Galaxy. This emission relies on thermal bremsstrahlung - where the trajectories of electrons are deviated as they fly by protons in the interstellar medium.

  Spinning dust emission from grains of dust in clouds which have a dipole induced across them (by strong electric fields), and spin in the presence of a magnetic field. We see spinning dust emission typically in photo dissociation regions (PDRs) such as Lambda Orionis (Harper et al., 2024).

  Thermal dust emission from warm dust (approximately 20 K or -250 °C) in the Galaxy. The radiation produced is that of a black-body radiator, and theoretically follows the Planck radiation law - although we make a modification to account for the distribution of different energies in the dust population.
- Spinning dust emission was only just discovered in 1997 (Leitch et al., 1997) and we have since been trying to understand its structure, it's spectrum and the mechanisms behind its emission.
- On small scales, we use techniques such as aperture photometry to probe individual structures in the night sky and obtain spectra.
- On large scales, we combine all-sky and half-sky maps with tools such as Commander to create maps of each foreground component.
- These component maps can then be used to remove foregrounds from CMB experiments, and create high-resolution, accurate maps of the CMB

A brief discussion of the mathematics of this problem sheet

We often look at diagrams like Figure 1, which shows how the brightness of each emission.

Whilst the exact functional form of each type of emission is quite complex, we often simplify things by assuming that synchrotron emission, free-free emission and thermal dust (at least at lower frequencies) scale exponentially with a constant index[1]. I.e. they scale as

$$S = A \times \nu_9^{\alpha}, \tag{1}$$

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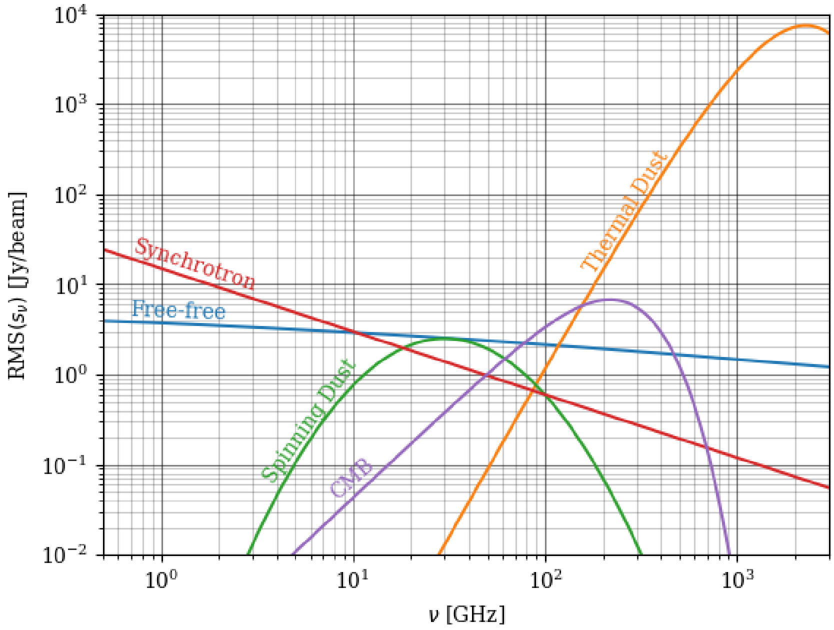
[1] This is adequate in most cases but is often untrue for supernova remnants, where radiative cooling (essentially the SNR radiating away its energy) can cause its synchrotron spectrum to steepen at higher frequencies (typically breaking above 10 GHz).

Figure 1: Sketch of typical RMS fluxes of foreground components at 5 $'$ resolution.



Note: This figure has been adapted from Rennie (2023). In reality, no source will look like this (since the high energies involved in processes leading to synchroton emission break down dust grains and therefore do not permit spinning dust emission to arise).

where $S(\nu)$ represents the source's integrated flux, ν represents its frequency in GHz[2], A represents the emission amplitude and α represents the spectral index for that specific component. Typical spectral indices (α values) for synchrotron emission, free-free emission and thermal dust emission are given in Table 1. Note firstly that no values are given for spinning dust emission - here we will not assume any particular shape and only discuss single flux measurements.

Table 1: Table of spectral indices for foreground emission components.

Name	Typical α
Synchrotron	-0.7
Free-free	-0.1
Thermal Dust	3.5

Note: All spectral indices are given with respect to flux density units (Janskys, Jy).

Section 1 will contain some problems to get you familiar with working in log-space. We will discuss supernova remnants and look at some examples. For section 2, we will explore the spectra of supernova remnants, and then finish with section 3 - which is based around the search for Anomalous Microwave Emission.

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[2] This is an arbitrary choice, but will make the numbers easier to handle later.

1 Linear algebra - Deriving the basics

Before we get stuck into some data, lets begin by getting familiar with working with exponential equations in log-log space.

A quick hint in these questions - if you're stuck, draw the emission like in Figure 1...

Question 1.1

I have a flux density measurement S_1 at frequency ν_1. The emission is dominated by a Galactic component with a spectral index α. Assuming the same component dominates at frequency ν_2, find an expression for the flux S_2 at ν_2.

Question 1.2

Now rearrange your answer to 1.1 so that the spectral index is now the subject.

Question 1.3

Use log laws to expand the logarithms in the numerator and denominator of your equation for α. Does this remind you of something from linear algebra?

You have now worked out how to go from two flux measurements to a simple exponential model... It might seems like a bit of a crude approximation, but now lets use it with some real data and show how powerful this simple tool is in analysing radio data.

2 Example: Supernova remnant

Before we dive into numbers, lets start by looking at maps of a real supernova remnant to see what we can deduce without even beginning to do any calculations. The images in Figure 2 show the supernova remnant W 44 as seen by the Effelsberg 100radio telescope at 2.7 GHz (Reich et al., 1984) and by the *Planck* satellite at 857 GHz (Planck Collaboration et al., 2020). In the low–frequency map we see a mix of synchrotron and free–free emission, wheras at 857 GHz emission from warm dust should dominate.

Question 2.1

Have a look at the shape of W44 - what can you see? Can you describe the features. We typically class supernovae as shells (which look like a complete or incomplete ring), filled structures (which typically look like small blobs as opposed to a shell or arc), or composites (where it looks like we might have some combination of the two). Which one do you think this is?

Compare the low-frequency map with the high-frequency map. What does this tell you about the composition of the W 44 complex.

Let us now turn our attention to a fictional supernova remnant called UBC 1 that you're observing with a radio telescope at 2 GHz and 4 GHz. You find that UBC 1 is 8.5 Jy at 2 GHz, and 6.0 Jy at 4 GHz.

Question 2.2

Find the spectral index and Flux at 1 GHz of the synchrotron emission based on your measurements, then write down an equation for the spectrum of synchrotron emission you detect.

The spectral index of synchrotron emission (α) may also be found by

$$\alpha = -\frac{p-1}{2}, \tag{2}$$

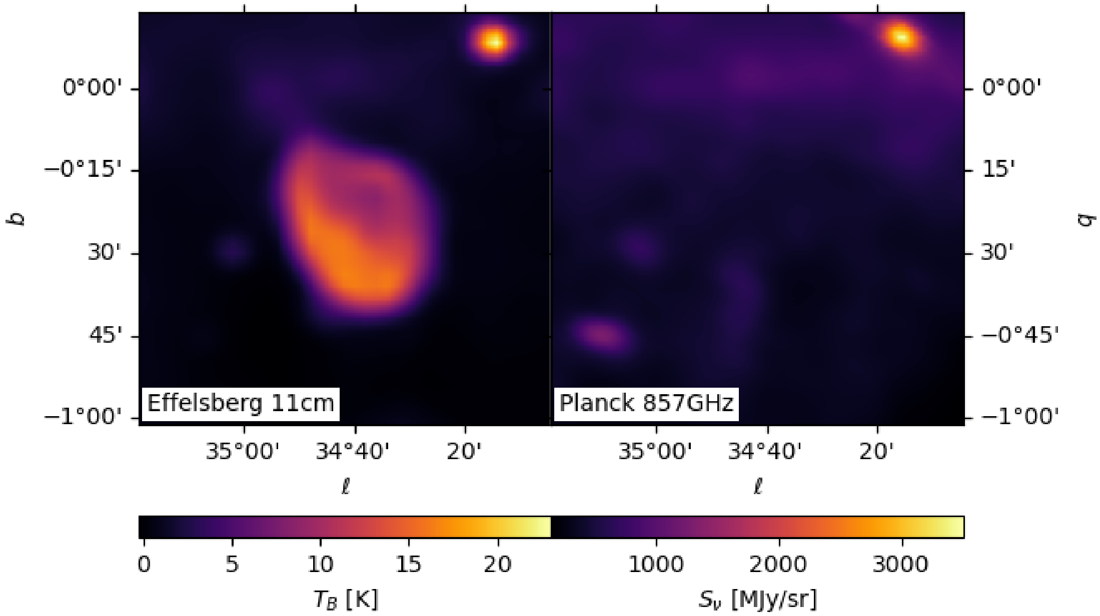


Figure 2: Effelsberg 100 m radio telescope image of the Westerhout 44 supernova remnant at 11 cm.

where p is the power-law index of the energy distribution of electrons that are the source of the synchrotron emission you see. A p-value of 2 is typical of a young supernova remnant, and this value can rise as the remnant ages.

Question 2.3

From your value for the spectral index, find the p-value and interpret it. What does it indicate about the age of the supernova remnant?

After discussing your results with your collegues, you discover that UBC 1 appears in public maps from a different telescope at 30 GHz. You find that at 30 GHz UBC 1 is 1.6 ± 0.1 Jy.

Question 2.4

Calculate what you would expect the flux at 30 GHz to be, assuming your model from question 2.2 is correct. Is it within 3 σ of your new measurement?

You find a 40 GHz map in the same data release, which finds UBC 1 to be 1.2 Jy.

Question 2.5

Calculate the spectral index between your colleague's points, does it match up with yours? Does it give you any more information about UBC 1's age?

3 Example: HII region

Lets now discuss a Hii region - which we'll call UBC 2. Again you come across UBC 2 in your data at 2 GHz and 4 GHz, where you measure it at 3.0 Jy and 2.8 Jy.

Question 3.1

Find the spectral index and flux at 1 GHz of free–free in UBC 2. From this information, present an equation for the spectrum of emission you detect.

Eager to learn about dust in this source, you find maps of UBC 2 at 500 GHz and 700 GHz. From this data you conclude that UBC 2 is 560 Jy at 500 GHz and 1815 Jy at 700 GHz.

Question 3.2

Assuming that thermal dust dominates the spectrum above 500 GHz and that in this frequency range the thermal dust spectrum can be approximated as an exponential with a spectral index of 2, find an equation for the thermal dust spectrum.

Question 3.3

From your answers to 3.1 and 3.2, find the frequency at which the contributions to the total SED from both thermal dust and free-free emission are equal.

Looking at UBC 2 in the 30 GHz map, you find a flux of 3.8 Jy.

Question 3.4

Calculate the flux you expect from UBC 2 at 30 GHz and compare it to your measurement. What fraction of the total emission at 30 GHz is free-free emission? Are there any conclusions you can draw?

In order to compare different observations of spinning dust emission, we calculate an emissivity ϵ – that being the ratio of excess emission detected to some tracer for the amount of dust. One such tracer is the total flux of the source at 545 GHz. Measuring excess at 30 GHz, this can be shown to be

$$\epsilon = 3.62 \times 10^4 \, \frac{\Delta S_{30}}{S_{545}} \; [\mu\text{K}/(\text{MJy}/\text{sr})] \tag{3}$$

where S_{545} represents the 545 GHz flux in Jy, and ΔS_{30} represents the excess emission observed at 30 GHz (i.e. the observed flux minus free–free and thermal dust).

Question 3.5

By calculating a flux at 545 GHz using your thermal dust model, find a value for the emissivity with respect to 545 GHz emission.
Planck observed this value to be (66 ± 7) μK/(MJy/sr) in the high-latitude sky (Planck Collaboration et al., 2016). Is your value consistent?

References

Harper S. E., et al., 2024, arXiv e-prints, p. arXiv:2405.04383

Leitch E. M., Readhead A. C. S., Pearson T. J., Myers S. T., 1997, ApJ, 486, L23

Planck Collaboration et al., 2016, A&A, 594, A25

Planck Collaboration et al., 2020, A&A, 641, A1

Reich W., Fuerst E., Haslam C. G. T., Steffen P., Reif K., 1984, A&AS, 58, 197

Rennie T. J., 2023, PhD thesis, Manchester, UK

4 Answers

Question 1: Linear algebra – Deriving the basics

1.1 Mapping S_1 to S_2

Let us begin with equation 1 rearranged for A,

$$A = \frac{S}{\nu^\alpha}.$$

Since we are assuming that at both frequencies the same component of foreground emission is dominant, A and α don't change. Therefore

$$A = \frac{S_1}{\nu_1^\alpha} = \frac{S_2}{\nu_2^\alpha}.$$

Now lets rearrange for S_2, which leaves us

$$S_2 = S_1 \left(\frac{\nu_2}{\nu_1}\right)^\alpha.$$

1.2 Finding α from $S = A\nu^\alpha$

As a strategy, lets try to isolate the α as a power of something. So lets start by saying

$$\frac{S_2}{S_1} = \left(\frac{\nu_2}{\nu_1}\right)^\alpha,$$

therefore

$$\ln\left(\frac{S_2}{S_1}\right) = \ln\left(\left(\frac{\nu_2}{\nu_1}\right)^\alpha\right).$$

We know that the $\log(a^b)$ is equal to $b\log(a)$, so lets apply it

$$\ln\left(\frac{S_2}{S_1}\right) = \alpha \ln\left(\frac{\nu_2}{\nu_1}\right).$$

Now a trivial rearrangement gives us

$$\alpha = \frac{\ln(S_2/S_1)}{\ln(\nu_2/\nu_1)}.$$

1.3 Parallels?

Expanding the logs give you

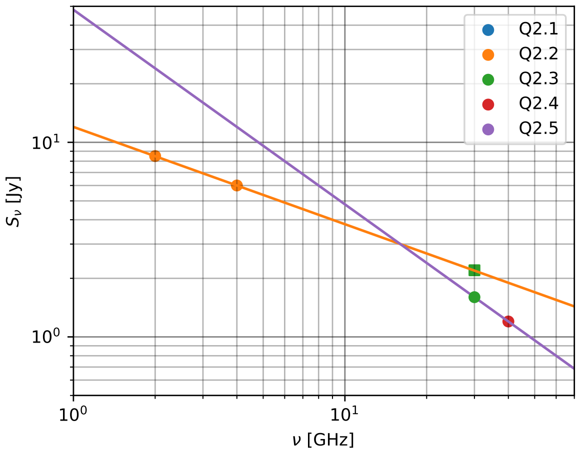
$$\alpha = \frac{\ln(S_2) - \ln(S_1)}{\ln(\nu_2) - \ln(\nu_1)} = \frac{\ln(S_1) - \ln(S_2)}{\ln(\nu_1) - \ln(\nu_2)}.$$

This looks like the standard equation for the gradient of a straight line

$$m = \frac{y_1 - y_2}{x_1 - x_2},$$

except that the x's and y's have been replaced by $\ln(\nu)$ and $\ln(S)$.

Question 2: Supernova Remnants



2.1 Finding structure and comparing maps

Here I'm looking for notes on shape and on the differences between the two maps. For a description of the shape, I'm after some discussion of the complex structure, some points to look at are;

- In the upper-right it looks like there might be a faint loop
- The lower-left bright structure looks like part of a shell maybe
- The structure is complicated
- The lower-left is significantly brighter than the upper right

As you're going round have a look at these descriptions and spark a conversation about them. – Do they have an idea what might cause this shape? Maybe something has disturbed the SNR shell as it has expanded. Maybe something is interacting with the shell in the bottom left and caused it to become brighter.

The main points I am after are as follows:

- There is no (or very little) structure in the high-frequency map (right) – there much be no dust
- The structure is quite complex, so it must be a composite supernova remnant

2.2 Fitting theory to measurement

From Question 1.2, we know how to get α. So

$$\alpha = \frac{\ln(S_2/S_1)}{\ln(\nu_2/\nu_1)} = \frac{\ln(8.5/6)}{\ln(2/4)} \approx \frac{0.348}{-0.693} \approx -0.5.$$

Note that in this equation, it doesnt actually matter what base you do the log to... So values of 0.151 and -0.301 (where you use $\log_{10}$) also work.

For our equation we also need a value for A, which we can find from either our answer to question 1.4 or a trivial rearrangement of Equation 1

$$A = \frac{S_1}{\nu_1^{\alpha}} = \frac{S_1}{\nu_1^{\frac{\ln(S_2/S_1)}{\ln(\nu_2/\nu_1)}}}.$$

Talking the simplest aproch (and using our value for α we get

$$A = \frac{8.5}{2^{-0.5}} = \frac{6}{4^{-0.5}} \approx 12 \text{ Jy}.$$

So our equation is

$$S = 12\nu^{-0.5} \text{ [Jy]}$$

2.3 Finding the electron energy power index

We have been given an equation for p, so lets rearrange. This gives us

$$p = 1 - 2\alpha,$$

which gives us

$$p = 1 - 2\alpha = 1 - (-1) = 2.$$

Therefore we can say that the p value indicates that UBC 1 might be a young supernova remnant.

2.4 Checking a high-frequency measurement

Using the answer to question 2.2,

$$S = 48\nu^{-0.5} = 48 \times 30^{-0.5} = 12 \times 0.183 \approx 2.2 \text{ Jy}.$$

Taking the difference between the two measurements and dividing by the error on our measurement we get that the measurement is approximately 0.6 Jy (or 6σ) below our model, and is therefore inconsistent.

2.5 Comparing spectral indices

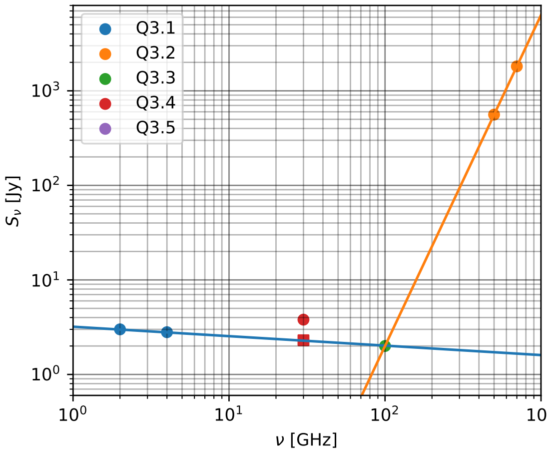
Repeating the steps above for finding a spectral index,

$$\alpha = \frac{\ln(S_2/S_1)}{\ln(\nu_2/\nu_1)} = \frac{\ln(1.6, 1.2)}{\ln(30/40)} \approx \frac{0.288}{-0.287} \approx -1.0.$$

This is much steeper than the -0.5 we got at lower frequencies.

Synchrotron spectra can break at higher frequencies due to radiative cooling having removed a significant amount of their energy energy. This process is quite slow - but provides more evidence for UBC 1 being a younger supernova remnant.

Question 3: HII Region



3.1 Free–free model

This question is effectively a repeat of 2.2:

$$\alpha = \frac{\ln(S_2/S_1)}{\ln(\nu_2/\nu_1)} = \frac{\ln(3/2.8)}{\ln(2/4)} \approx \frac{0.0689}{-0.693} \approx -0.1,$$

$$A = \frac{3}{2^{-0.1}} = \frac{2.8}{4^{-0.1}} \approx 3.2 \text{ Jy}.$$

Making our equation

$$S = 3.2\nu^{-0.1} \text{ [Jy]}$$

3.2 Thermal dust model

Again a repeat of 3.1:

$$\alpha = \frac{\ln(S_2/S_1)}{\ln(\nu_2/\nu_1)} = \frac{\ln(560/1815)}{\ln(500/700)} \approx \frac{-1.176}{-0.336} \approx -3.5,$$

$$A = \frac{560}{500^{-3.5}} = \frac{1815}{700^{-3.5}} \approx 2.0 \times 10^{-7} \text{ Jy}.$$

Making our equation

$$S = 2.0 \times 10^{-7}\nu^{3.5} \text{ [Jy]}.$$

3.3 Thermal dust and free–free equivalence

Setting the answers to 3.1 and 3.2 as equal

$$S = 3.2\nu^{-0.1} = 2 \times 10^{-7}\nu^{3.5},$$

$$1.6 \times 10^7 = \nu^{3.6},$$

$$\nu = (1.6 \times 10^7)^{1/3.6},$$

$$\nu = 100 \text{ GHz}.$$

Running this through either emission model gives that both emission mechanisms contribute 2 Jy to the total flux.

3.4 Extrapolating to 30 GHz

30 GHz is sufficiently below 100 GHz that free-free must be more dominant than thermal dust. Starting from the answer to 3.1, we know that

$$S = 3.2\nu^{-0.1}\ [\text{Jy}],$$

therefore substituting in numbers we obtain that

$$S = 3.2 \times 30^{-0.1} = 2.3\,\text{Jy}.$$

Furthermore, we can calculate from 3.2 that the contribution of thermal dust is 0.02 Jy confirming that it is subdominant.

Since the measurement at 30 GHz is 3.8 Jy, the difference (excess) is 1.5 Jy. So the free–free emission makes up 61 % of the total emission observed at 30 GHz.

Since there is excess emission reported, and the flux calculated from low-frequency data is consistent with free–free emission, we can infer that this is a detection of spinning dust emission.

3.5 Calculating emissivity

From 3.2 we find that the flux at 545 GHz is

$$S = 2 \times 10^{-7}\nu^{3.5} = 756\,\text{Jy}.$$

Our excess is 1.5 Jy, and so our emissivity must be

$$\epsilon = 3.62 \times 10^{4}\,\frac{1.5}{756} = 72\,\mu\text{K}/(\text{MJy}/\text{sr}). \qquad (4)$$

This is consistent with the *Planck* value for the high–latitude sky.