Physics Circle Oct 16 with Dr. Valery Milner

Question 1:

The Physics Olympics are starting!

Up first, the hammer throw. You watch as a student spins around a 5kg ball at the end of a 2m long chain. It is travelling at 2m/s and is looking to be a record throw. Right before they release the ball, however, their grip slips and an extra half meter of chain is let out (the ball is now at the end of a 2.5m chain).

Key concept:

> *Angular momentum is essentially linear momentum ($p = mv$) constrained to rotation about an axis. Considering a point mass rotating about an axis like above, it is calculated as $L = mrv$, where m is mass, r the radius of rotation, and v its tangential velocity. Like linear momentum, it is conserved in a system with no net force (or in this case torque) as follows from Newtons laws.*

As a physicist, you can't help but measure these things and are quick to calculate the new speed of the ball. You assume the ball can be modelled as a point mass rotating in a circle, and that no net forces are present in the system.

a) What do you find its new speed to be?

Bonus How does the kinetic energy of the ball change? Think about where it might go.

Unfortunately, the throw didn't turn out to be record-breaking, but as the second event approaches, you turn your physics brain toward figure skating.

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You find the skater to have a height of 1.5m, mass of 100kg, and wingspan of 1.5m with their arms outstretched. Realizing it is more of a stretch to model the skater as a point-mass, you take additional steps to ensure accuracy in your calculations.

Key concept:

*Moment of inertia (I) is a measurement of resistance to changes in rotational motion, like how mass measures a resistance to changes in linear motion. The full formula for angular momentum is $L = Iw$ where I is the moment of inertia (kg*m^2) and w is angular velocity (radians/sec).*

b) Modelling the skater as a cylinder, what is their moment of inertia?

Bonus: Could you more closely approximate the skater's moment of inertia? Make reasonable assumptions about body proportions.

c) If they rotate at 2 revolutions per second, what is their angular momentum?

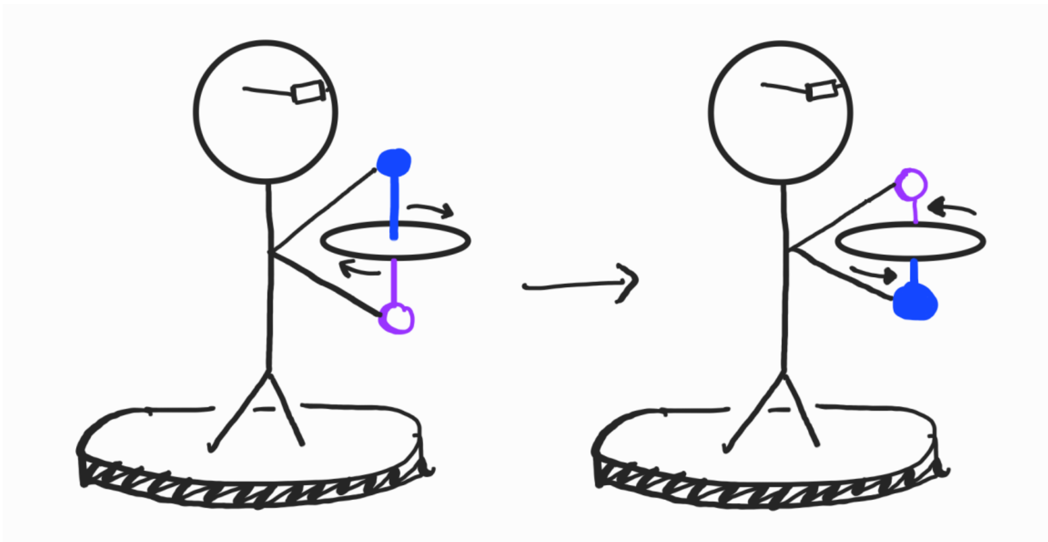
d) The next movement requires them to pull their arms into their body, reducing their wingspan to just 0.5m. What is their new angular velocity?

Problem 3: Spinning wheel and conservation of angular momentum

Consider a spinning wheel: for simplicity we assume the wheel is uniform. As we saw in question 1, we need to calculate the moment of inertia of the uniform wheel/disk to find its angular momentum. After doing some integral calculus which we will skip, one can show a disk of **radius R** and **total mass M** has the following moment of inertia:

$$I_{Disk} = \frac{1}{2}MR^2$$

Albert is holding a spinning **disk of mass 2 kg and radius 0.5 meters** which is rotating at an angular velocity of 5 rotations per second. **He is standing on a platform which can rotate without friction.**



a) Convert 5 rotations per second to the unit of radians per second. (360 degree = 2π radians) In the left side image, from Albert's perspective the wheel is rotating in the **clockwise direction**. Using the right hand rule, find out if the direction of the angular momentum is upwards or downwards with respect to the ground. If you are not familiar with the right hand rule, call a volunteer and we can explain!

b) Using the result from the previous part and the formula for moment of inertia of a disk, calculate the angular momentum of this rotating wheel. Assuming upwards to be positive direction and dowards to be negative (just a matter of definition), **calculate the angular momentum of the disk**. Be sure to include the sign (positive or negative).

c) Albert then flips the disk slowly without letting it go. This is shown in the right side of the figure above. The wheel keeps spinning but since it is flipped upside down, it now rotates **counter-clockwise**. What changes about the angular momentum of the disk?

d) Now we consider the combined system of Albert and the disk. If the angular momentum of the disk has changed and the total angular momentum has to be conserved, what happens to Albert who is standing on a platform that can frictionlessly rotate?

e) Assume that Albert is a cylinder, with **mass 50kg** and radius **0.5 m.** The moment of inertia of a cylinder has the same formula as a disk, $I_{cylinder} = 1/2\, MR^2$. Calculate what velocity Albert must be rotate in.

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Bonus: Cross products and Angular momentum

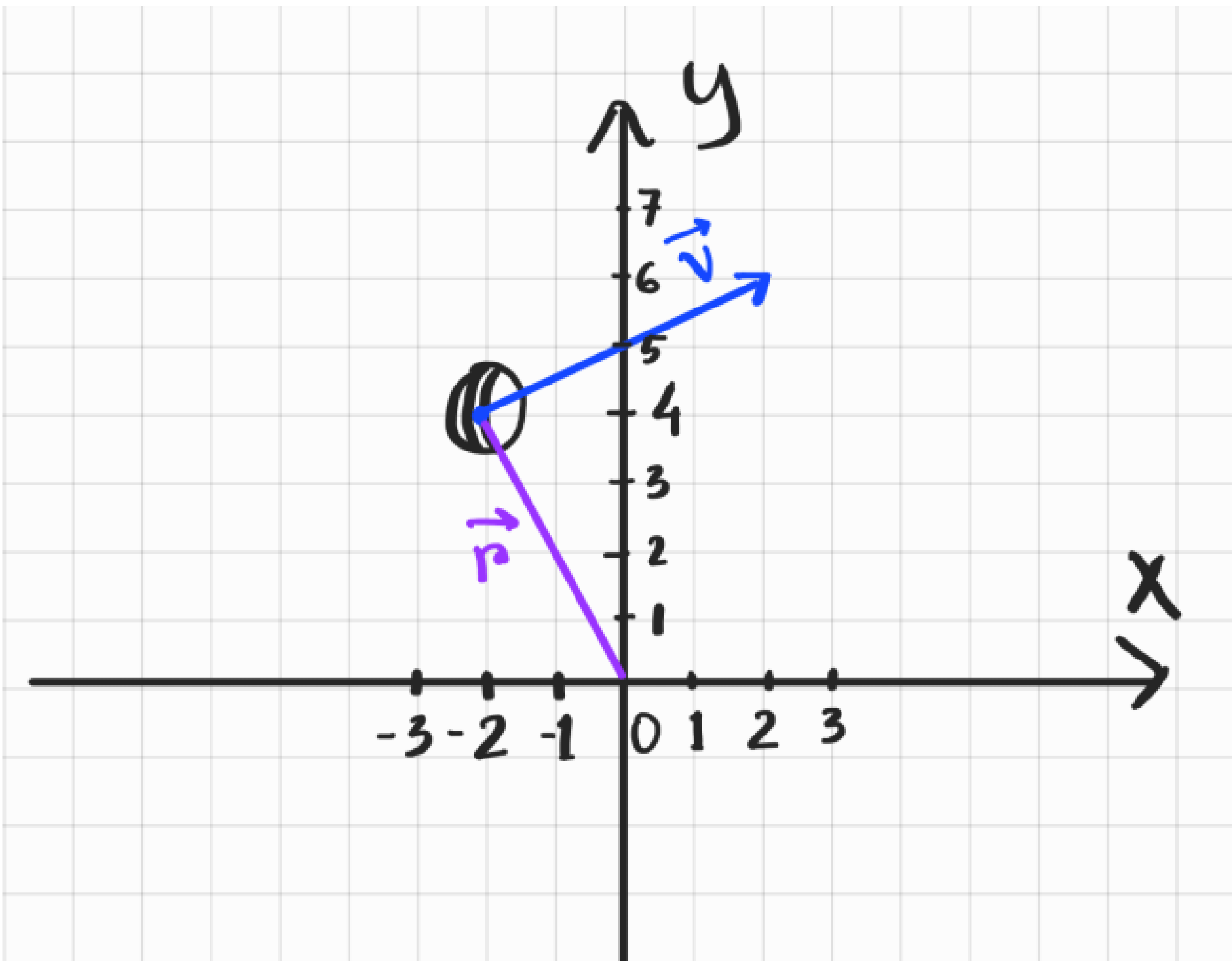
Angular momentum is vector, and we can try to calculate it using vector cross products. LEt us start with a short refresher/summary of cross products. Cross products are a way to multiply two vectors. The magnitude of a cross product is defined as:

$$\left|\vec{A} \times \vec{B}\right| = |A|\,|B|\,sin\theta$$

Where θ is the angle between the vectors A and B.

We can generalize the formula $L = mrv$ to:

$$L = m\,\vec{r} \times \vec{v}$$



In the figure above a ball is shown at a certain position defined by a coordinate system. The velocity vector is labelled in blue and the position vector is labelled in purple, identify the values of the vectors $\vec{r}$ & $\vec{v}$. Assuming the ball is 0.1 kg, calculate the magnitude of angular momentum.