Physics Circle Question Set

November 20, 2024

Part 1

1. **Calcium Ion Movement**
   What physical principles govern the movement of calcium ions in the cell?
   Hint: Consider diffusion and electric forces in charged particles.

2. **Calcium Pumps**
   The cell membrane has calcium pumps that actively transport Ca^{2+} out of the cell. If each pump removes 10 ions per second, estimate how many ions a cell needs to remove in one minute if there are 1,000 pumps. How does this process impact the concentration gradient of calcium ions over time?

3. **Calcium Concentration Gradient**
   Cells often have varying calcium ion concentrations inside and outside the membrane. If the concentration inside is 10^{-4} mol/L and outside is 10^{-3} mol/L, calculate the ratio of external to internal concentration. Why might this gradient be important for signaling?

4. **Diffusion Distance**
   Calcium ions diffuse through the cytoplasm at a rate determined by the diffusion coefficient $D = 0.1\,\mu\text{m}^2/\text{ms}$. The approximate distance calcium ions travel (x) in a given time (t) is:

   $$x = \sqrt{2Dt}$$

   Question: If $D = 0.1\,\mu\text{m}^2/\text{ms}$, calculate how far the calcium ions travel in $t = 10\,\text{ms}$.
   Hint: Convert μm to meters if needed.

5. **Energy and Work**
   Calcium pumps use ATP molecules to transport ions. If one ATP molecule provides 10^{-19} J of energy, and each pump requires 3×10^{-19} J to move one Ca^{2+}, calculate:
   (a) How many ATP molecules are needed to remove 10^6 calcium ions?
   (b) What is the total energy required to remove 10^6 ions?

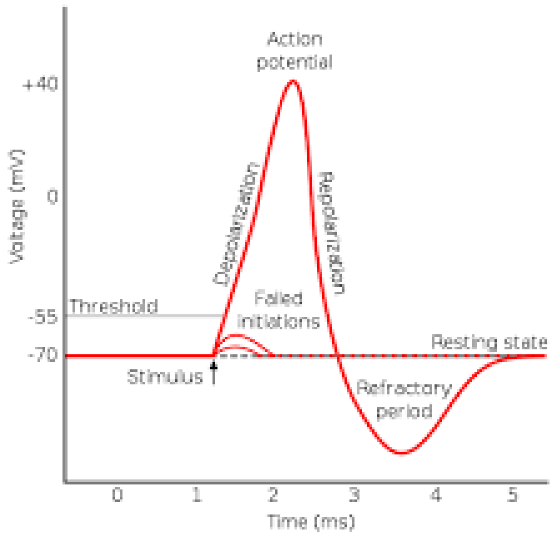
6. **Membrane Potential and Electric Field**
   The cell membrane generates an electric potential difference of 70 mV across a thickness of 5 nm.
   Question: What is the electric field strength across the membrane?
   Hint: Use:
   $$E = \frac{V}{d}$$
   where $V = 70 \times 10^{-3}$ V and $d = 5 \times 10^{-9}$ m.

7. **Force on Calcium Ions**
   Each calcium ion has a charge of $q = 2e$, where $e = 1.6 \times 10^{-19}$ C.
   Question: What is the force exerted on a calcium ion by the electric field calculated in the previous question?
   Hint: Use:
   $$F = qE$$

8. **Challenge: Signal Propagation in Cells**
   (a) If calcium signals must reach the farthest part of a cell, how might the cell size affect the balance between diffusion and active pumping? For instance:
      - Would larger cells need faster pumps or higher diffusion coefficients to maintain effective signaling? Why?
      - How might a long, thin cell (like a neuron) differ in its calcium signaling requirements compared to a spherical cell?
   (b) Consider a hypothetical cell with calcium pumps that are twice as efficient as normal pumps. How might this affect the range and speed of calcium waves? Could over-efficiency ever harm signaling?

Part 2

Neurons in your body send signals with electrical impulses, or action potentials. These action potentials are caused by a shift in the electrical potential difference across the neuron's cell membrane, caused by ions flowing out of the cell via ion channels. The ion channels (in the case of the neuron, mostly sodium and potassium channels) are voltage-gated, meaning they open and close depending on the voltage difference across the cell membrane. The action potential follows this graph:



The membrane can be modeled as a parallel resistor-capacitor (RC) circuit. To build up the potential difference, we must first charge the capacitor. The equation for charging a capacitor in an RC circuit is:

$$V = E\left(1 - e^{-\frac{t}{\tau}}\right)$$

where V is the potential difference of the capacitor, E is the electromotive force, t is time, and τ is the time constant ($\tau = R \cdot C$). This is the time needed for the capacitor to charge to 67 percent of its maximum.

Given typical electrical values of a neuron:

$$E = 100\,\text{mV},\ R = 1\,\text{k}\Omega,\ C = 1\,\mu\text{F}.$$

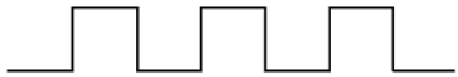
1. **Charging the Capacitor:** How long does it take to charge the capacitor to 90% of its maximum value? Sketch V over time as the capacitor charges.

2. **Discharging the Capacitor:** Once the power input is taken away, the capacitor will discharge according to:

   $$V = V_0 e^{-\frac{t}{\tau}}$$

   How long does it take to discharge to 10% of its maximum value? Sketch V as the capacitor discharges.

3. **Signal with Impulses:** What might the signal of the capacitor look like if you send in impulses? Sketch V over time if the input signal is a square wave.



For detailed reading on the biophysics of action potentials, see:
`https://neurophysics.ucsd.edu/courses/physics_171/single_cell_dynamics.pdf`