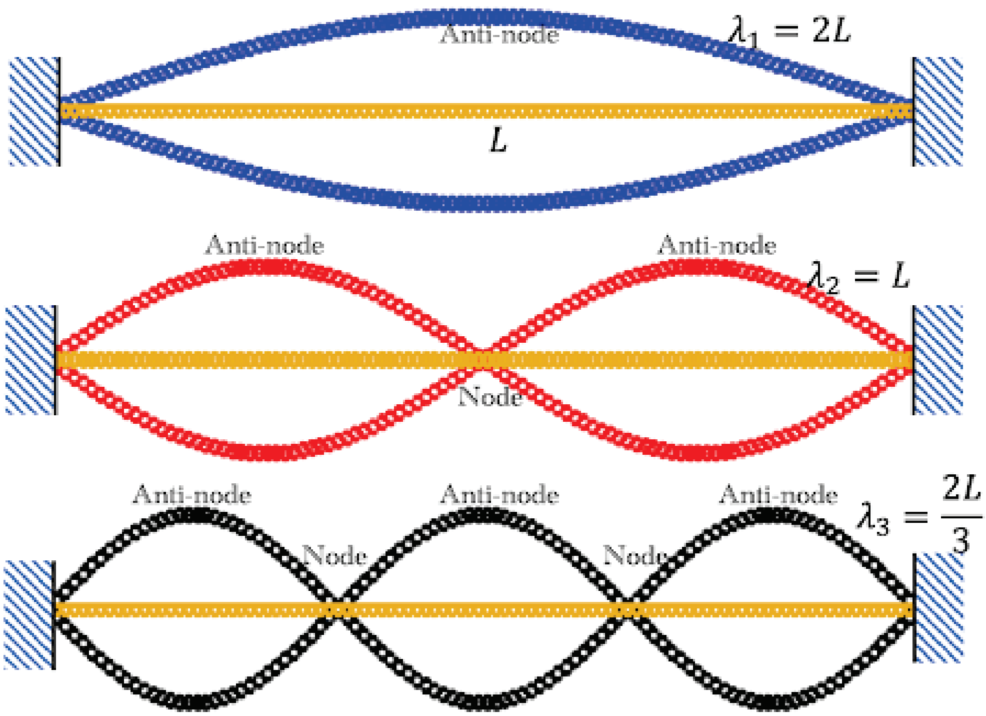
Solution

Part 1: Modes of Oscillation

When a string is fixed at both ends, it can vibrate in different patterns called "modes." Each mode has a different number of "loops" along the length of the string, where each loop represents a part of the string that moves up and down while the ends stay fixed.

Understanding the Harmonics

1. **First Mode (Fundamental Mode):** The string vibrates in one single loop between the two fixed ends, creating the simplest pattern. The distance from one end to the other is half a wavelength for this mode. 2. **Second Mode:** The string has two loops, each representing half a wavelength, so the total length of the string now equals a full wavelength. 3. **Third Mode:** The string has three loops, meaning the total length of the string is now one and a half wavelengths.



Wavelength Patterns

The wavelength λ in each mode relates to the length L of the string as follows:

- **Fundamental Mode (1 loop):** $\lambda = 2L$
- **Second Mode (2 loops):** $\lambda = L$
- **Third Mode (3 loops):** $\lambda = \frac{2L}{3}$

In general, for the n-th mode, the wavelength λ_n is:

$$\lambda_n = \frac{2L}{n}$$

where n is the number of loops.

Part 2: Speed of Wave Propagation

The speed v of a wave on a stretched string depends on the tension T and the mass per unit length μ (linear mass density). The formula for the wave speed v is:

$$v = \sqrt{\frac{T}{\mu}}$$

Given Values

- Tension $T = 100\,\mathrm{N}$ - Mass per unit length $\mu = 0.01\,\mathrm{kg/m}$

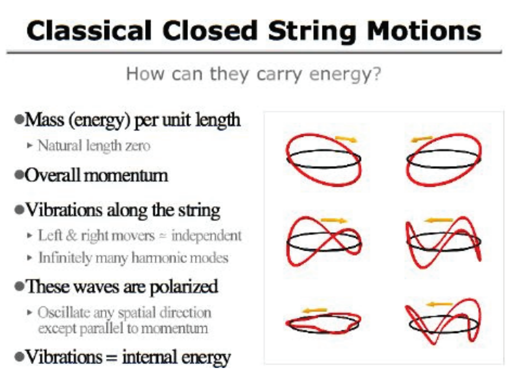
Calculation

$$v = \sqrt{\frac{100}{0.01}} = \sqrt{10000} = 100\,\mathrm{m/s}$$

Thus, the speed of the wave on the string is $100\,\mathrm{m/s}$.

Bonus: Connection to String Theory

The concept of modes of oscillation on a string under tension has a fascinating parallel in string theory. In string theory, fundamental particles are modeled as tiny, vibrating strings. Different modes of vibration (analogous to the harmonics in this problem) correspond to different physical properties of particles, such as mass, charge, and spin. Just as each harmonic on the string has a distinct frequency, each mode of vibration in string theory represents a unique "state" of the string, which can be interpreted as a different type of particle.



Solutions

Part A:
As a 4D hypersphere passes through our 3D space, the object that we would observe is a 3D sphere. The "cross-section" of the hypersphere in 3D space follows a similar pattern to the 2D world's experience with the 3D sphere:

- First, the 4D hypersphere touches our 3D space at a point.
- As it enters, you would observe a growing 3D sphere. It expands as the hypersphere's 4D volume intersects with our 3D world.
- The sphere reaches its maximum size at the midpoint of the hypersphere's passage through our world.
- Finally, the 3D sphere shrinks back to a point as the hypersphere exits 3D space.

So, you would see a 3D sphere grow and shrink as the hypersphere moves through our 3D world.

Part B:

A 4D cube (or tesseract) has:
1. 16 vertices: Each vertex can be described by a set of 4 0s and 1s, giving 2^4 = 16 possible combinations.
2. 32 edges: Each vertex is connected to four other vertices, leading to 32 edges.
3. 24 square faces: Using the number of edges, we can determine there are 24 faces.]
4. 8 cubic cells: These cubic cells are the 3D analogs of the square faces of a cube.

When the tesseract passes through our 3D space, we would observe a 3D projection or "shadow" of the tesseract. This would likely look like a changing shape, with additional vertices, edges, and faces appearing and disappearing as the tesseract moves through our world.

If we were able to "slice" the tesseract and look at its 3D cross-sections, we would see objects such as cubes and distorted versions of cubes, depending on the position of the tesseract relative to our 3D space. The number of visible edges and faces would change dynamically as the tesseract moves.