Modes of Oscillation and Speed of Wave on a String

UBC PHYSICS CIRCLE

Question

Imagine a string of length L that is fixed at both ends. The string is under tension T and has a mass per unit length μ. You are tasked with determining the modes of oscillation and the speed of waves traveling along the string.

Part 1: Modes of Oscillation

The modes of oscillation on the string are given by standing waves. These waves occur at specific frequencies, corresponding to the harmonic modes of the string. The general form of the displacement for the n-th harmonic mode is:

$$y_n(x,t) = A_n \sin\left(\frac{n\pi x}{L}\right)\cos(\omega_n t)$$

where:

- A_n is the amplitude of the n-th mode,
- n is the mode number ($n = 1, 2, 3, \ldots$),
- L is the length of the string,
- ω_n is the angular frequency of the n-th mode.

Determine the wavelength λ_n for the n-th mode and explain how it relates to the length of the string L. As a hint, draw the different modes and think about how wavelength is related to length.

Part 2: Speed of Wave Propagation

The speed v of a wave traveling along the string is determined by the tension T and the mass per unit length μ. The formula for the speed of the wave is:

$$v = \sqrt{\frac{T}{\mu}}$$

Using this formula, calculate the speed of a wave on the string if the tension is $T = 100\,\mathrm{N}$ and the mass per unit length is $\mu = 0.01\,\mathrm{kg/m}$.

Bonus: Connection to String Theory

The principles behind the oscillation of a string under tension serve as a simple model for the more complex vibrating strings in string theory. In string theory, fundamental particles are conceptualized as vibrating strings, where different modes of vibration correspond to different particle properties, such as mass and charge. Think about how the harmonics of the string could relate to these physical properties.

Question 2

Imagine that you are a 2D creature living on the surface of a sphere, which we will call the "2D world." You can only perceive two dimensions — length and width — and you are unaware of any "depth" in your environment. Now, imagine a 3D sphere (a "ball" in 3D space) passes through your 2D world. From your perspective as a 2D creature, you can't see the full 3D sphere, but only the part of it that intersects with your 2D plane.

Part A: Draw what you would see from your perspective as the 3D sphere moves through your 2D world:

1. When the 3D sphere is far away.

2. As the sphere is moving through your world, up to the moment right before it starts receding.

3. As the sphere exits your world.

Now, let’s imagine a similar scenario, but this time with a 4D sphere (also called a "hypersphere"). You live in 3D space, but the 4D sphere exists in four-dimensional space, which you cannot directly perceive.

If a 4D hypersphere passes through our 3D world, we can reason about how its 3D "cross-section" will change in a way similar to how a 3D sphere's "cross-section" changes for the 2D creatures.

Describe the shape and behavior of the object as it enters, passes through, and exits our 3D world.

Part B: Now consider a 4D cube, also called a tesseract. Just as a cube is a 3D object with vertices, edges, and faces, a tesseract exists in four dimensions and has a more complex structure.

1. How many vertices or corners would a tesseract have? See if you can find a relationship between the number of vertices and the dimension for a square and a cube.

2. How many edges does a tesseract have? Think about how many other vertices each vertex is connected to.

3. How many faces does a tesseract have? Try to use your previous two answers to reason through this.

4. A line segment has 2 vertices, a square has 4 edges, and a cube has 6 faces. What is the 4D analogy to this, and how many of these things does a tesseract have?