Standard Model Problem Set

January 15th, 2025

1. Beta Decay

Beta decay is a radioactive process where a neutron in a nucleus is converted to a proton, releasing an antineutrino and an electron (beta-minus decay), or a proton is converted into a neutron, releasing a neutrino and a positron (beta-plus decay).This occurs when a nucleus has too many protons or neutrons to be stable and is caused by the weak interaction.

The neutrino was suggested by Pauli in 1931 as a way to explain the conservation of mass-energy, linear, and angular momentum in beta decay. However, it was only observed and verified in 1956.

(a) Write the complete nuclear equations for the types of beta decay. Examples:

$$H_1^3 \xrightarrow{\text{12.3 years}} He_2^3 + e_{-1}^0 + \bar{\nu}_0^0$$

(tritium) (beta particle) (antineutrino)

i. Beta-minus decay of Actinium-230.

$$^{230}_{89}\mathrm{Ac} \rightarrow {}^{0}_{-1}\mathrm{e} + {}^{230}_{90}\mathrm{Th} + \bar{\nu}$$

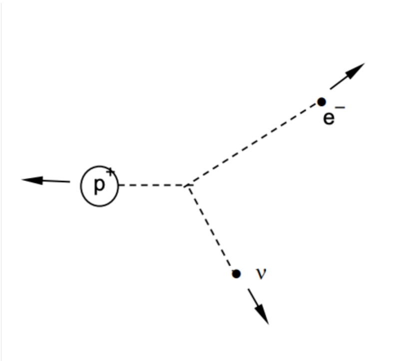
ii. Beta-plus decay of Neptunium-234.

$$^{234}_{93}\text{Np} \rightarrow {}^{0}_{+1}\text{e} + {}^{234}_{92}\text{U} + \nu$$

iii. Beta-minus decay of Calcium-45.

$$^{45}_{20}\text{Ca} \rightarrow {}^{45}_{21}\text{Sc} + {}^{0}_{-1}\text{e} + {}^{0}_{0}\nu$$

(b) Consider a neutron at rest decaying into a proton, electron, and antineutrino as shown below. The electron is moving at a 30 degree angle to the horizontal at 1.30×10^6 m/s and the proton is moving at 4.5×10^3 m/s.. Neutrinos can have varying levels of energy in beta decay depending on how much of the released energy goes to the beta particle. What is the momentum of the neutrino, and what is its angle to the horizontal? Assume the neutrino has zero mass (this is not accurate).



Some helpful constants:

- Mass of a proton: 1.67×10^{-27} kg,
- Mass of an electron: 9.11×10^{-31} kg,

Momentum is conserved in each dimension. The initial momentum in each dimension is zero (the neutron is at rest), so the final momentum in each dimension must also add up to zero.
x dimension: $m_p v_p = m_e v_e \cos(30) + p_{nx}$
y dimension: $m_e v_e \sin(30) = p_{ny}$

$$p_{nx} = \left(1.67 \cdot 10^{-27}\,\text{kg} \cdot 4.5 \cdot 10^3\,\text{m/s}\right) - \left(9.11 \cdot 10^{-31}\,\text{kg} \cdot 1.3 \cdot 10^6\,\text{m/s} \cdot \cos(30°)\right) = 6.49 \cdot 10^{-24}$$

$$p_{ny} = 9.11 \cdot 10^{-31}\,\text{kg} \cdot 1.3 \cdot 10^6\,\text{m/s} \cdot \sin(30°) = 5.92 \cdot 10^{-25}$$

$$p_n = \sqrt{(p_{nx})^2 + (p_{ny})^2} = 6.51 \cdot 10^{-24}$$

$$\theta = \arctan\left(\frac{p_{ny}}{p_{nx}}\right) = 5.2°$$

2. Time Dilation

Time dilation is a phenomenon predicted by Einstein's theory of relativity, where time appears to pass slower for an object moving at a speed close to the speed of light compared to an observer at rest. This effect is described by:

$$\Delta t' = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where:

- Δt is the time interval measured in the rest frame of the moving object,
- $\Delta t'$ is the dilated time (time interval measured in the stationary observer's frame),
- v is the relative velocity of the moving object,
- c is the speed of light (3×10^8 m/s).

High-energy muons are created in Earth's upper atmosphere during cosmic ray interactions. The average lifetime of a muon at rest is about 2.2 μs (microseconds). Muons are typically created 10 km above Earth's surface and travel at close to light speed (99.8% of speed of light or 0.998c).

(a) Disregard any effects of relativity. Calculate how far muons traveling at this speed would travel. Would they reach Earth's surface if created 10 km above?

A muon travels at $0.998c = 2.994 \times 10^8$ m/s. We can put these values into the classical formula for distance

$$d = vt = 2.994 \times 10^8 \times 2.2 \times 10^{-6} = 658 \text{ m}$$

So, the muon would only travel 658 meters on average and not reach Earth's surface

(b) Account for time dilation using the formula provided. Assuming the muon travels at the same speed for the dilated time ($\Delta t'$), calculate how far it would travel. Does it now reach Earth's surface?

$$\Delta t' = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{2.2 \times 10^{-6}}{\sqrt{1 - 0.998^2}} = 49.2\ \mu\text{s}$$

So, in the frame of people on Earth, the muon actually survives 49.2 microseconds instead of 2.2 microseconds. So, in our reference frame, we see the muon travel a distance of:

$$d = v \times (\Delta t') = 2.994 \times 10^8 \times 49.2 \times 10^{-6} = 14.7\text{km}.$$

As a result, in our reference frame, we see the muon travel a total distance of 14.7km on average. So, muons that are created at the top of the atmosphere (10km above) are frequently detected on Earth's surface.

3. Parity Violation

Imagine you're looking in a mirror. The reflection of your right hand looks like a left hand in the mirror, but everything else appears to behave the same way. Parity is the idea that the laws of physics should work the same way in this "mirror image" world. Scientists used to believe this was true for all forces of nature—until the cobalt-60 experiment proved otherwise.

In this experiment, scientists studied a type of radioactive atom called cobalt-60. These atoms decay by releasing electrons. The cobalt-60 nuclei were cooled and placed in a magnetic field, which caused the atoms to align in a specific way, like all spinning tops pointing in one direction(spins are polarized). These electrons are emitted along the negative z direction.

If you were to flip the atom's polarized spin and hence flip the direction of the tops spinning, what direction would you expect the emitted electrons to go? The answer might surprise you!

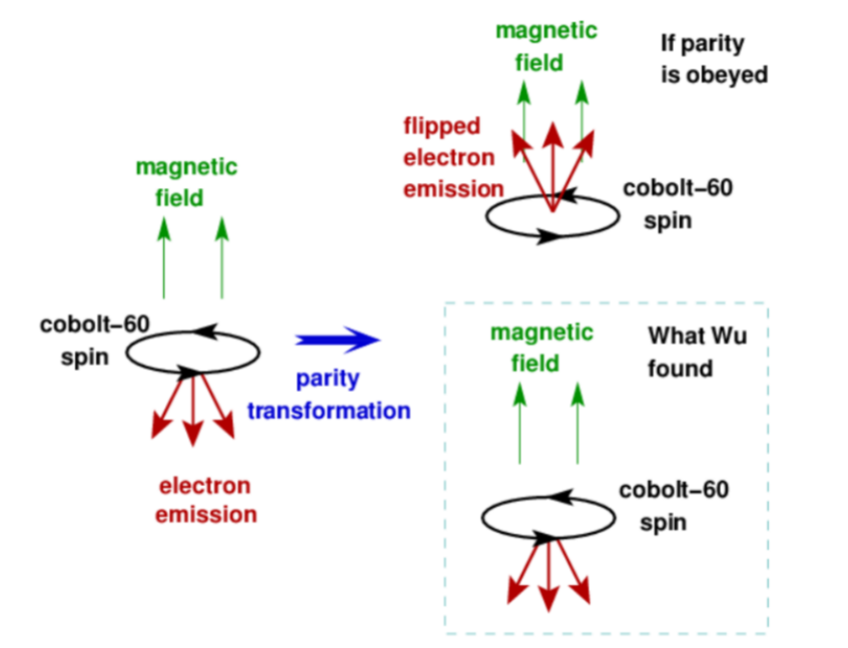


Image and extract source : [Doublexscience]

If parity were conserved, electrons would be emitted evenly in both directions when you imagine the system in a mirror. But what scientists found was shocking: the electrons were not emitted evenly. Most of them preferred to go in one direction (downward relative to the nuclei's "spin"). This meant the "mirror world" would look different from the real world—parity was violated! This discovery was groundbreaking because it showed that the weak nuclear force (which governs this kind of radioactive decay) does not follow the symmetry of parity, unlike other forces like gravity or electromagnetism.