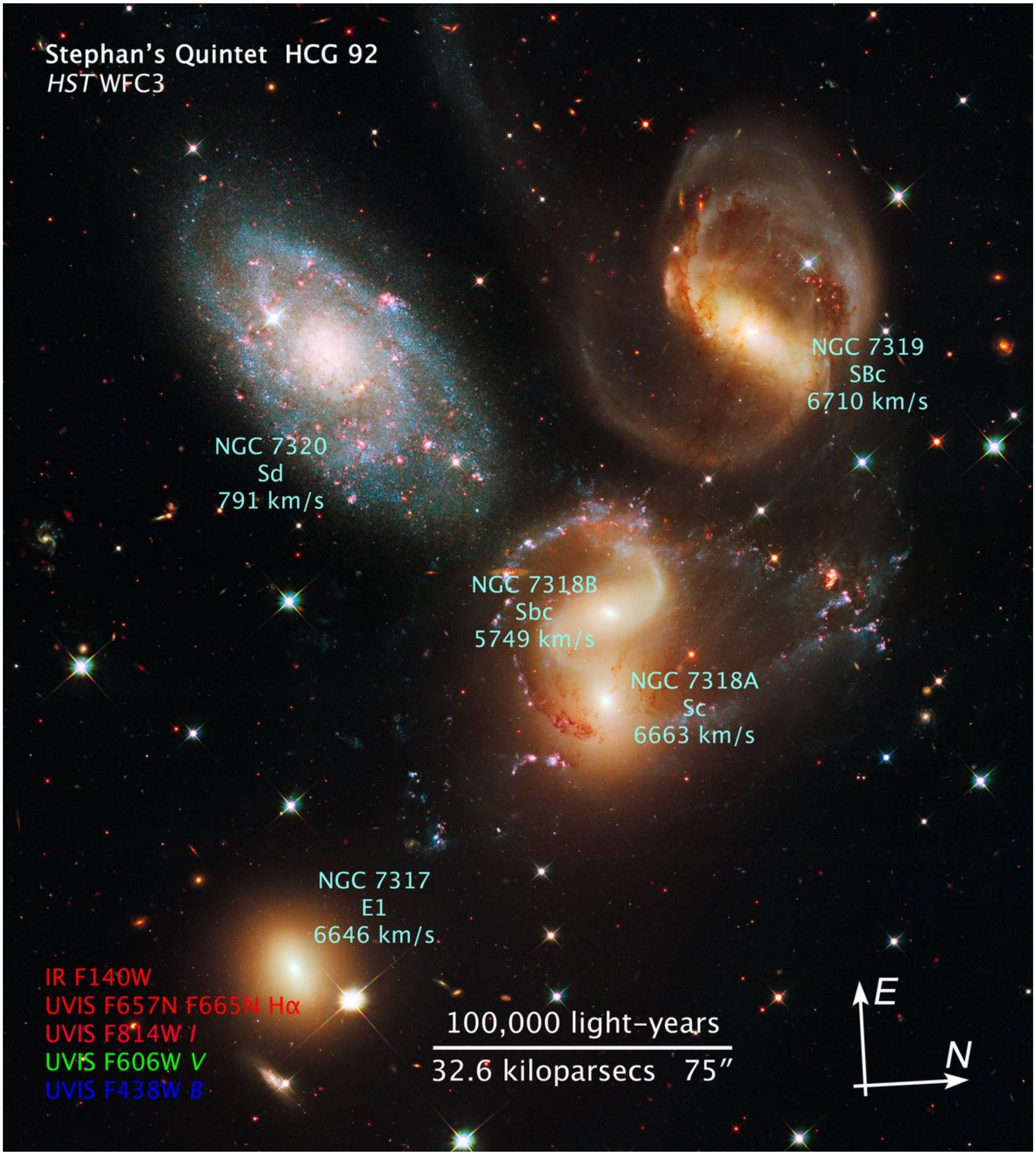
Physics Circle Problem Set November 5, 2025



1. Identifying a Physical Group

Stephan's Quintet is one of the most stunning visual groupings of galaxies. However, to consider the galaxies to be in a physical group, we need to evaluate how close they are. The rough requirements are that the galaxies in a group should be less than 2 MPc (mega parsecs) away from each other and the velocity difference should be less than or around $1000\,\mathrm{km\,s^{-1}}$. Based on this information and the above figure, which galaxies belong to the same physical group?

2. Redshift and Velocity Relation

The velocities of these galaxies describe how fast they are moving away from us. This is often called the *line-of-sight velocity*, and is related to the redshift of the galaxy (z) by the equation:

$$v = cz$$

where c is the speed of light.

1. Calculate the redshift of these galaxies.
2. In searching for protocluster candidates at high redshift, we usually set the criterion of redshift difference of the candidates to be approximately $\Delta z < 0.1$. Does this make sense to you? If not, why do you think we choose this redshift difference?

3. Virialization of the Group

In astrophysics, the *virial mass* is the mass of a gravitationally bound astrophysical system, assuming the virial theorem applies. In the context of galaxy formation and dark matter halos, the virial mass is defined as the mass enclosed within the virial radius of a gravitationally bound system—a radius within which the system obeys the virial theorem.

The main equation is:

$$M_{\mathrm{vir}} \approx \frac{3R\,\sigma^2}{G}$$

Where $G \approx 6.67428 \cdot 10^{-11}\mathrm{m^3kg^{-1}s^{-1}}$ (or, in different units $G = 4.3 \times 10^{-6}\,\mathrm{kpc\,km^2\,s^{-2}\,M_\odot^{-1}}$). Assuming that each galaxy in Stephan's Quintet has a mass of $5 \times 10^{10}\,M_\odot$ (solar masses), R is the radius of the system, σ is the velocity dispersion (the root-mean-square velocity of the galaxies relative to the system's mean velocity), and G is the gravitational constant, evaluate if the physical group you identified in Problem 1 is virialized.

4. Hubble's Law and Distance Estimation

Hubble's Law relates a galaxy's recession velocity to its distance from us:

$$v = H_0\, d$$

where H_0 is the Hubble constant.

1. Using the velocities of the galaxies, estimate their distances assuming $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.
2. Compare the distances you find with the MPc grouping criterion. Do these distances support your identification of which galaxies are physically associated?

Solution

1. Identifying a Physical Group

NGC 7320 does not belong to the group. Although it is close to the other galaxies in the image, its velocity is much smaller, indicating that this galaxy is much closer to us.

2. Redshift of the galaxies:

$$\begin{aligned} \text{NGC 7320:} &\quad z = 0.002637 \\ \text{NGC 7319:} &\quad z = 0.022366 \\ \text{NGC 7318A:} &\quad z = 0.01916 \\ \text{NGC 7318B:} &\quad z = 0.02221 \\ \text{NGC 7317:} &\quad z = 0.02215 \end{aligned}$$

From this calculation, we can see that the redshift difference of galaxies in local clusters is typically very small—on the order of ~ 0.005. However, when searching for protocluster candidates at high redshift, we usually set the redshift difference criterion to be $\Delta z \sim 0.1$.

There are two main reasons for this:

1. When galaxies are in protoclusters that have not yet been bound by gravity, they can have a wider range of relative velocities. It will take time for them to settle into the cluster and share a common velocity.
2. Observational constraints: at high redshift, the best achievable uncertainties in redshift measurements can be as large as ~ 0.1, especially when only imaging data are available. Therefore, it is not meaningful to consider redshift differences smaller than the uncertainty limit.

3. Virial Mass Estimate:

Given:

$$R = 49\,\mathrm{k}Pc, \quad \sigma = 6454.5\,\mathrm{km\,s^{-1}}, \quad G = 4.3 \times 10^{-6}\,\mathrm{kpc\,km^2\,s^{-2}\,M_\odot^{-1}}$$

The virial mass is:

$$M_{\rm vir} = 1.42 \times 10^{15}\, M_\odot$$

The total mass of the galaxies, based on our estimations, is only $5 \times 10^{10}\, M_\odot$. This is much smaller than the virial mass.

This suggests that the galaxies should be gravitationally bound, since the virial mass represents the maximum mass that can be bound by gravity within a certain radius. However, we may be underestimating the actual masses of the galaxies. More importantly, we have neglected the mass of the gas and dark matter halos. These components must be measured more carefully to determine whether the system is truly gravitationally bound.

Question 4: Hubble's Law and Distance Estimation

Using Hubble's Law,

$$v = H_0 d \quad \Rightarrow \quad d = \frac{v}{H_0},$$

and $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, with $v = cz$ and $c = 3 \times 10^5\,\mathrm{km\,s^{-1}}$:

$$\begin{aligned}
\text{NGC 7320:} \quad & v = 3 \times 10^5 \times 0.002637 = 791\,\mathrm{km\,s^{-1}} \\
& d = \frac{791}{70} = 11.3\,\mathrm{Mpc} \\
\text{NGC 7319:} \quad & v = 3 \times 10^5 \times 0.022366 = 6710\,\mathrm{km\,s^{-1}} \\
& d = \frac{6710}{70} = 95.9\,\mathrm{Mpc} \\
\text{NGC 7318A:} \quad & v = 3 \times 10^5 \times 0.01916 = 5748\,\mathrm{km\,s^{-1}} \\
& d = \frac{5748}{70} = 82.1\,\mathrm{Mpc} \\
\text{NGC 7318B:} \quad & v = 3 \times 10^5 \times 0.02221 = 6663\,\mathrm{km\,s^{-1}} \\
& d = \frac{6663}{70} = 95.2\,\mathrm{Mpc} \\
\text{NGC 7317:} \quad & v = 3 \times 10^5 \times 0.02215 = 6645\,\mathrm{km\,s^{-1}} \\
& d = \frac{6645}{70} = 94.9\,\mathrm{Mpc}
\end{aligned}$$

Interpretation:

NGC 7320 is clearly much closer to us (11 Mpc) compared to the others (82 M to 96 Mpc). This confirms that NGC 7320 does **not** belong to the physical group, while NGC 7317, NGC 7318A/B, and NGC 7319 are all located at roughly the same distance and are physically associated within the same group, well within the 2 Mpc grouping criterion.